Solving QSAT in Sublinear Depth

Alberto Leporati • Luca Manzoni • Giancarlo Mauri
Antonio E. Porreca • Claudio Zandron

P systems here

P systems here

Uniform families of P systems

P systems here

Uniform families of P systems

with **active membranes**

P systems here

Uniform families of P systems

with **active membranes**

with **charges**

P systems here

Uniform families of P systems

with **active membranes**

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& weak **non-elementary division** rules

P systems here

Uniform families of P systems

with **active membranes**

with **charges**

& weak **non-elementary division** rules

Depth is Important

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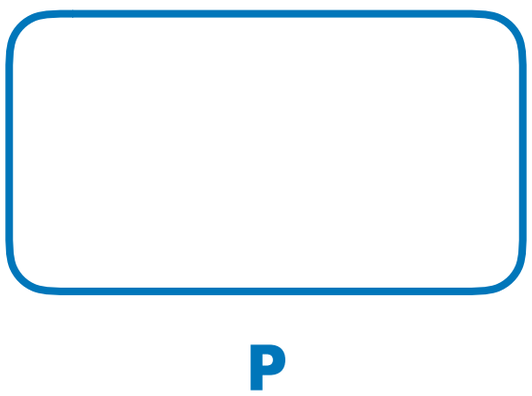
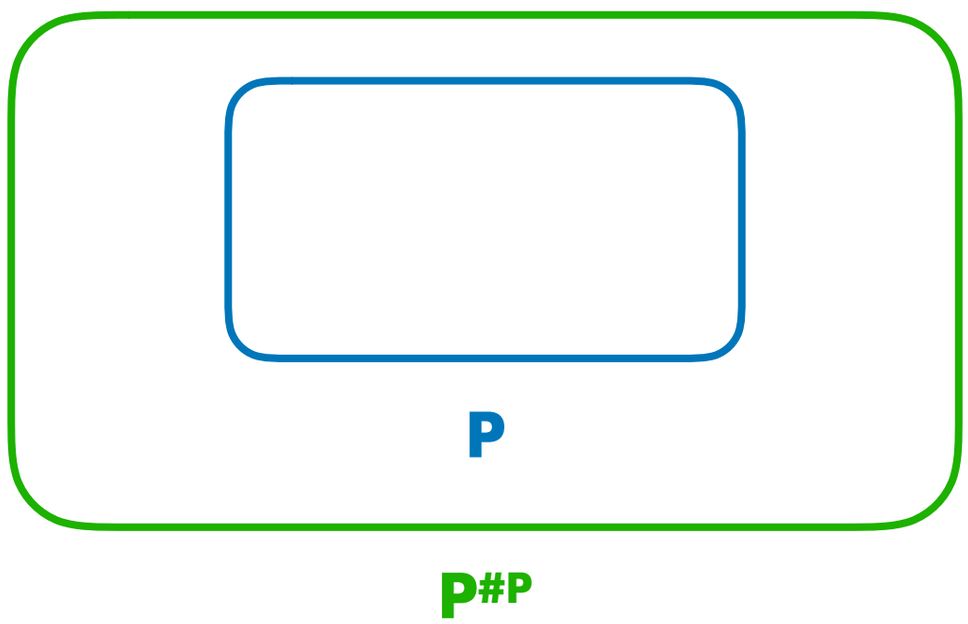


Depth is Important



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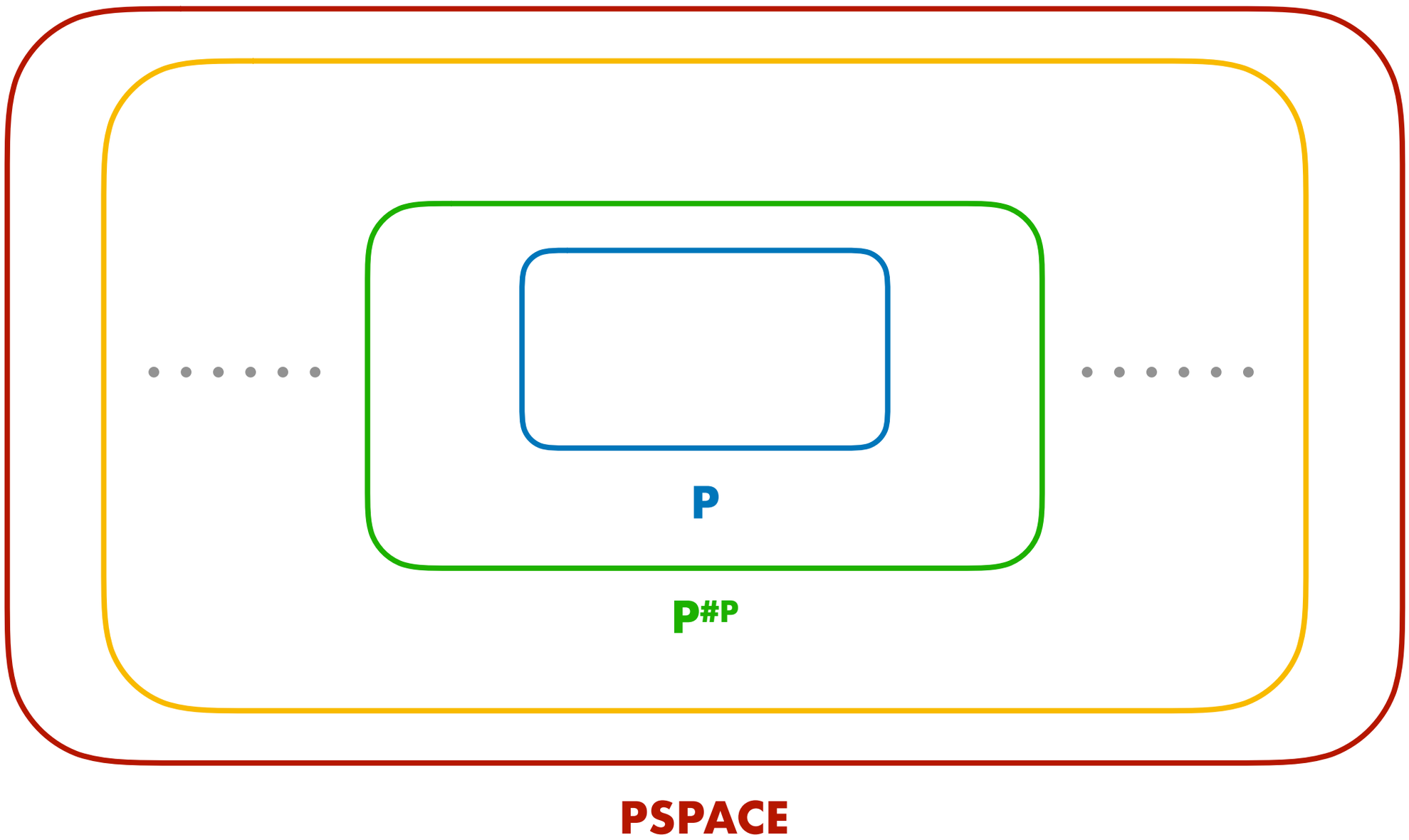
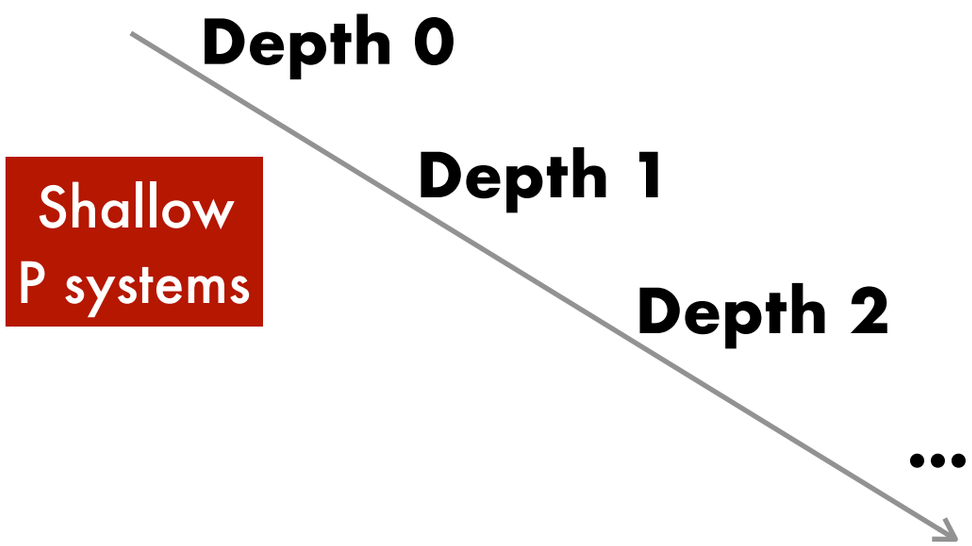


“The Gap”

"The Gap"



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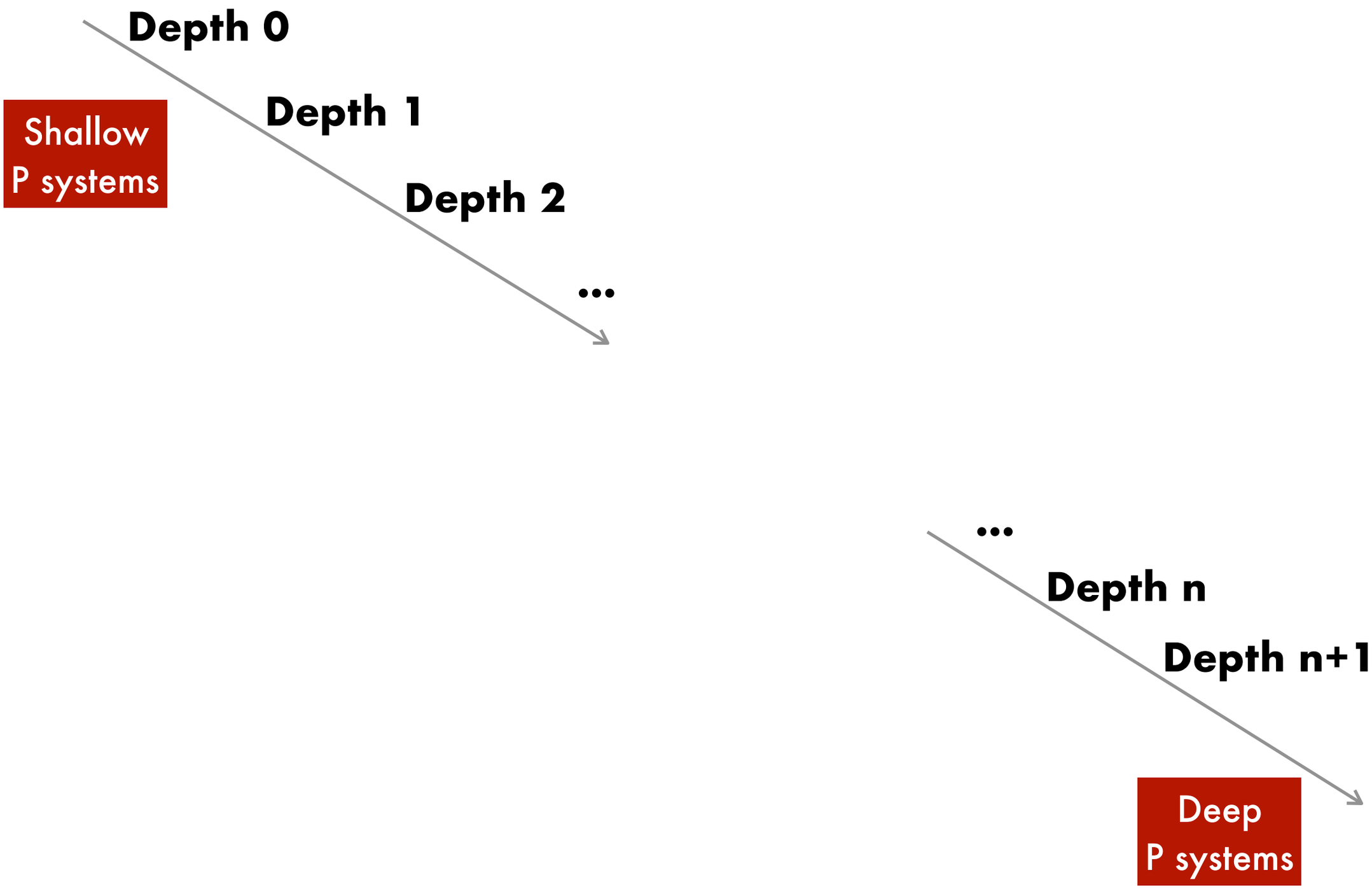
Depth 0
Depth 1
Depth 2
...
Shallow
P systems
...
Depth n
Depth n+1
Deep
P systems

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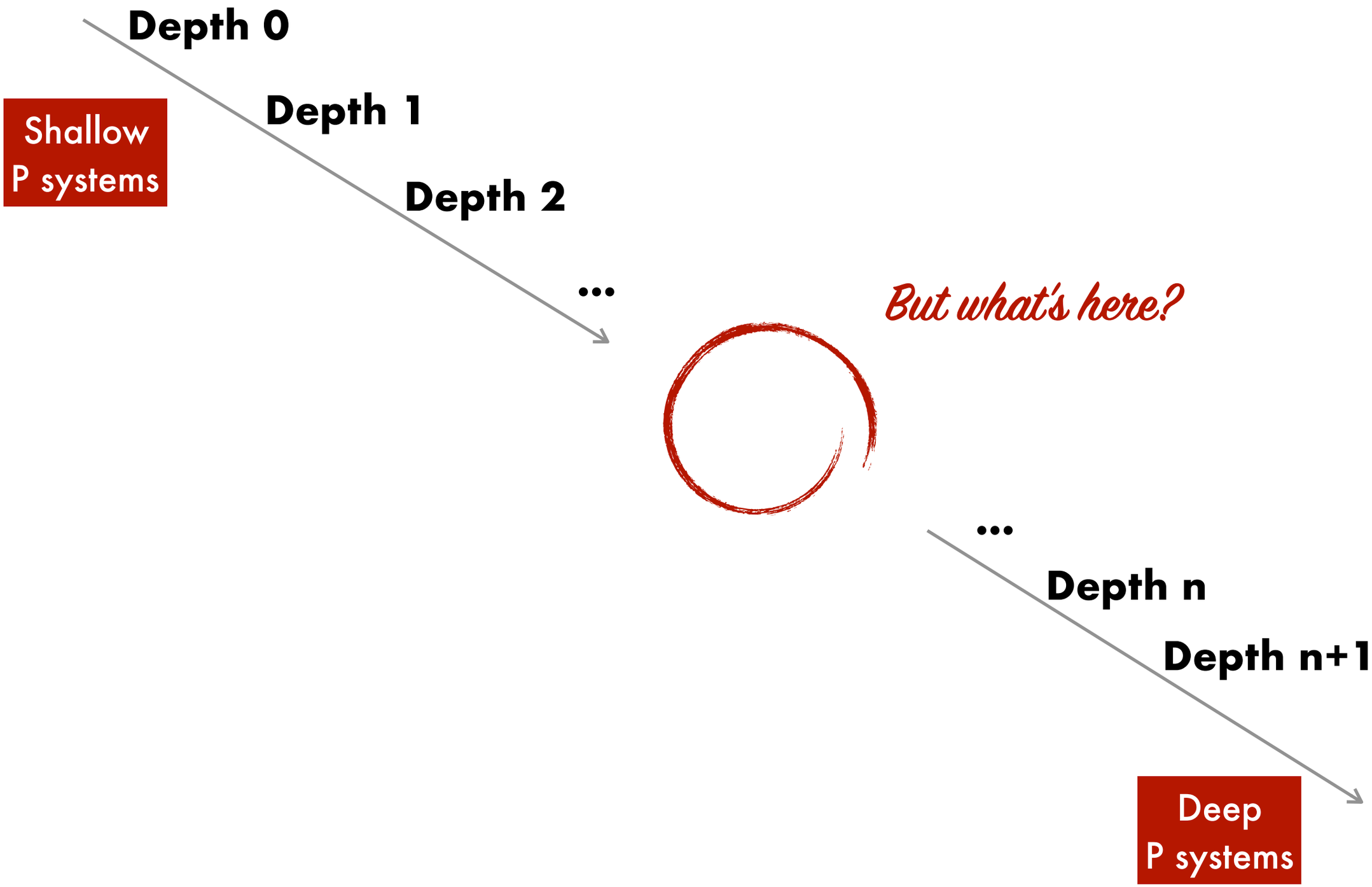
Depth 0
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But what's here?
...
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P systems

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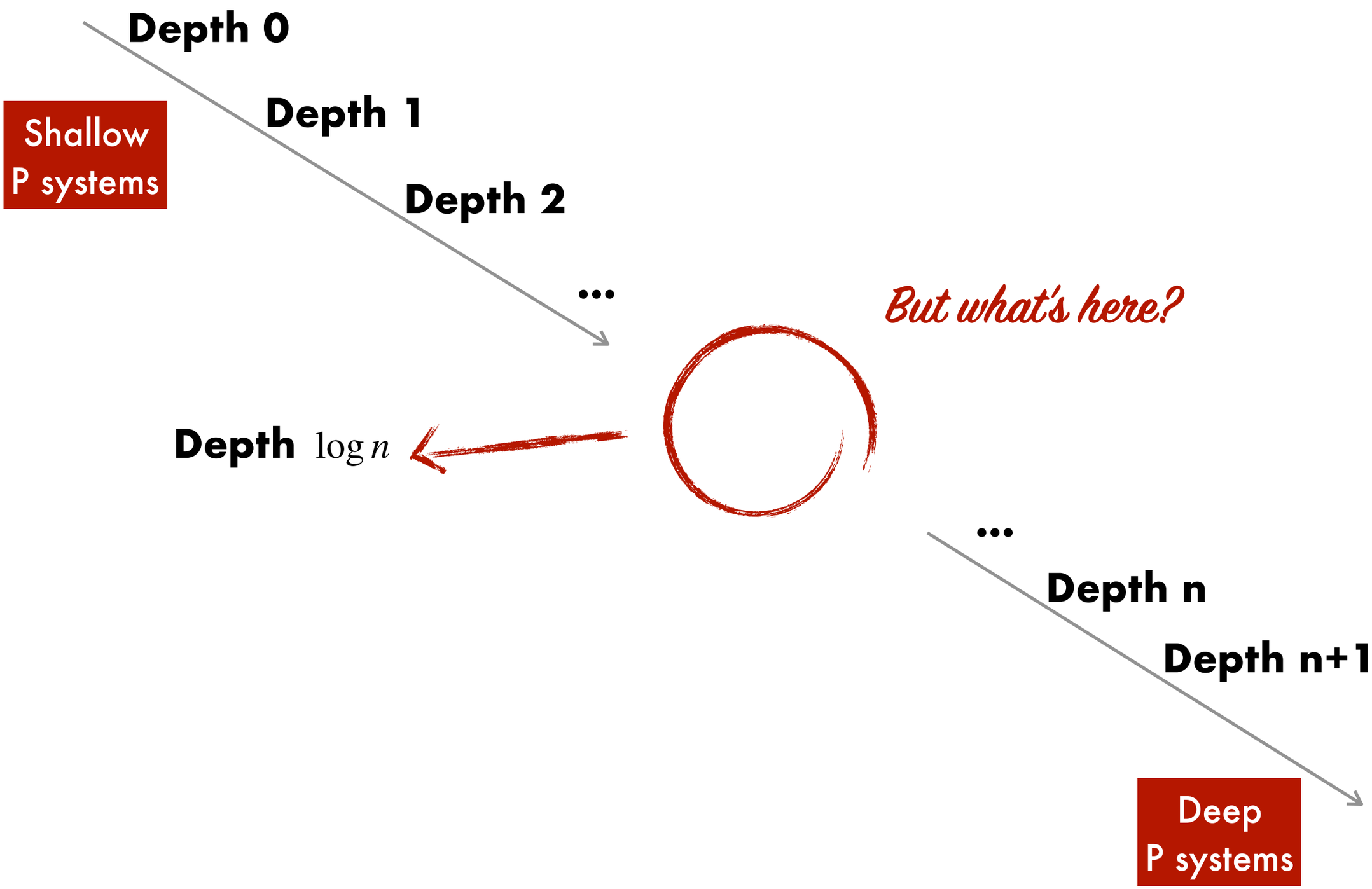
Depth 0
Depth 1
Depth 2
...
Shallow
P systems
But what's here?
Depth $\log n$
...
Depth n
Depth n+1
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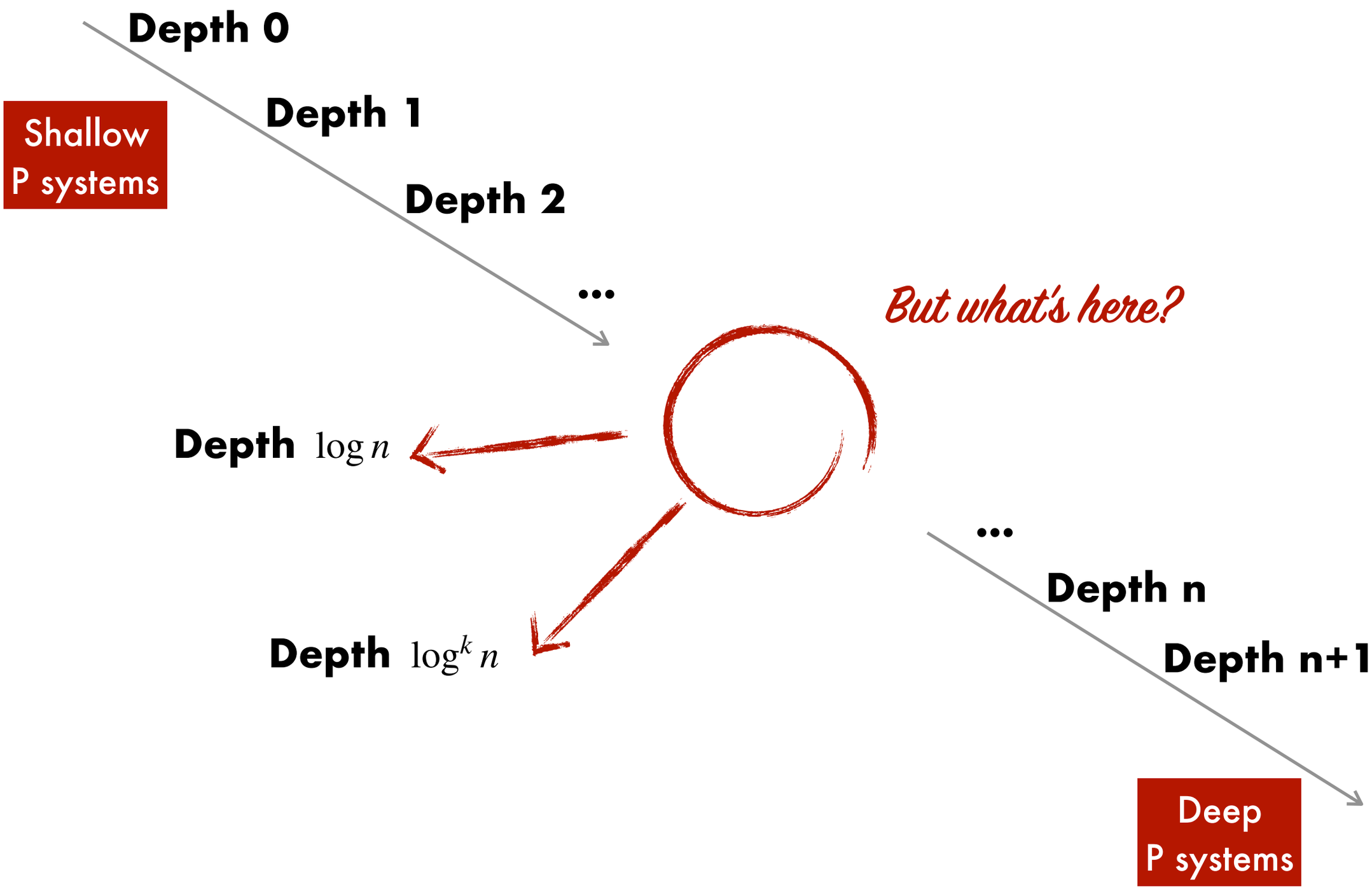
"The Gap"


Depth 0
Depth 1
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P systems
But what's here?
Depth $\log n$
Depth $\log^k n$
...
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Depth n+1
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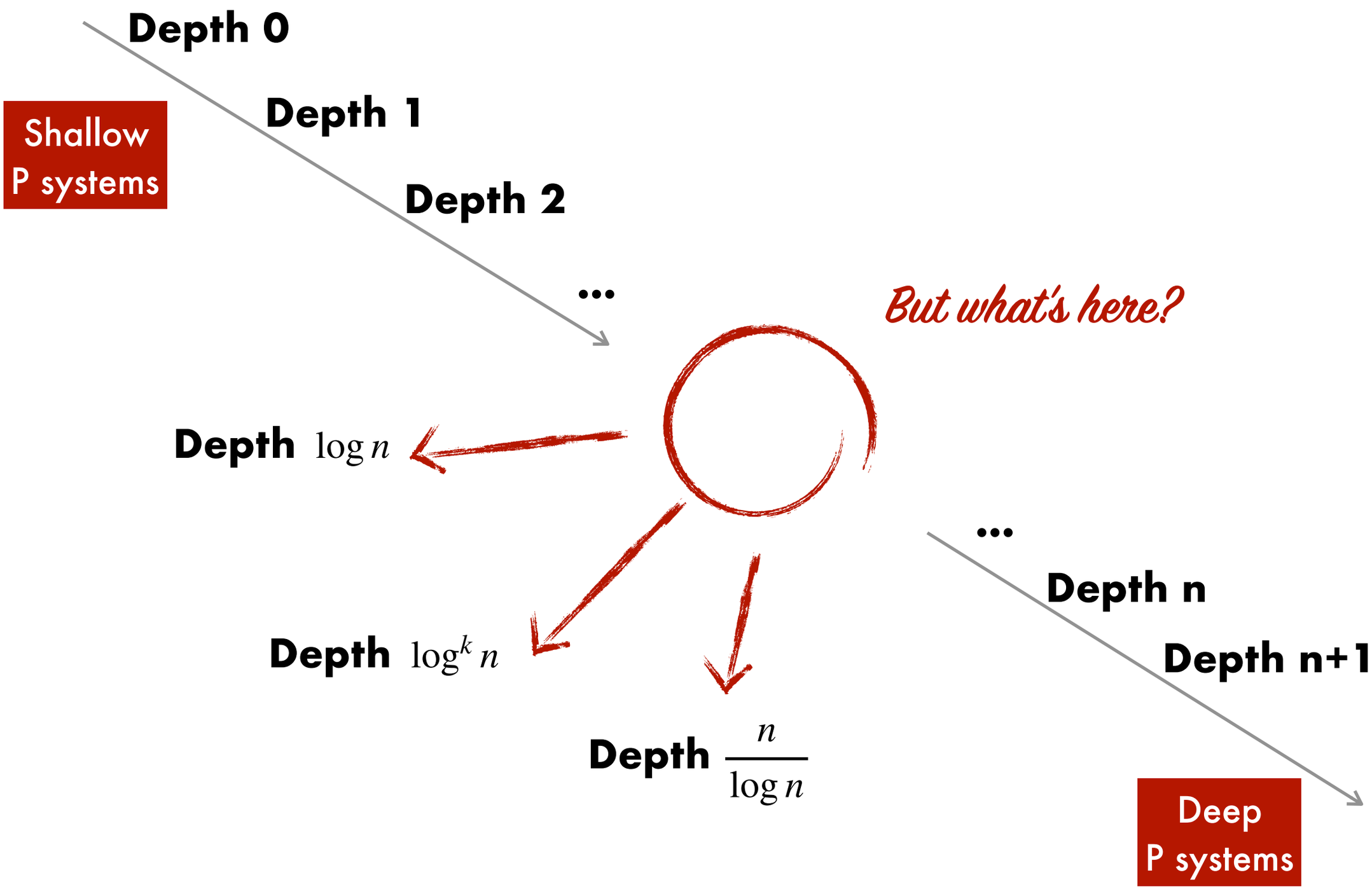
P systems

"The Gap"


Depth 0
Depth 1
Depth 2
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Shallow
P systems
But what's here?
Depth $\log n$
Depth $\log^k n$
Depth $\frac{n}{\log n}$
...
Depth n
Depth n+1
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QSAT

$$\exists x_1 \forall x_2 \exists x_3 \; (x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_1 \vee x_2 \vee x_3)$$

QSAT

$$\exists x_1 \forall x_2 \exists x_3 \; (x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_1 \vee x_2 \vee x_3)$$

n quantifiers

QSAT

n variables

Fully quantified

$$\exists x_1 \forall x_2 \exists x_3 \; (x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_1 \vee x_2 \vee x_3)$$

n quantifiers

QSAT

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$$\exists x_1 \forall x_2 \exists x_3 \; (x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_1 \vee x_2 \vee x_3)$$

n quantifiers

m clauses

QSAT

n variables

Fully quantified

$$\exists x_1 \forall x_2 \exists x_3 \; (x_1 \vee x_2 \vee \neg x_3) \wedge (\neg x_1 \vee x_2 \vee x_3)$$

n quantifiers

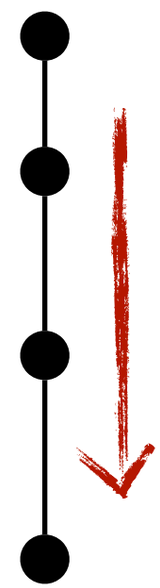
m clauses

Question: **Is the formula valid?**

The Standard Approach

The Standard Approach

- Generate all assignments



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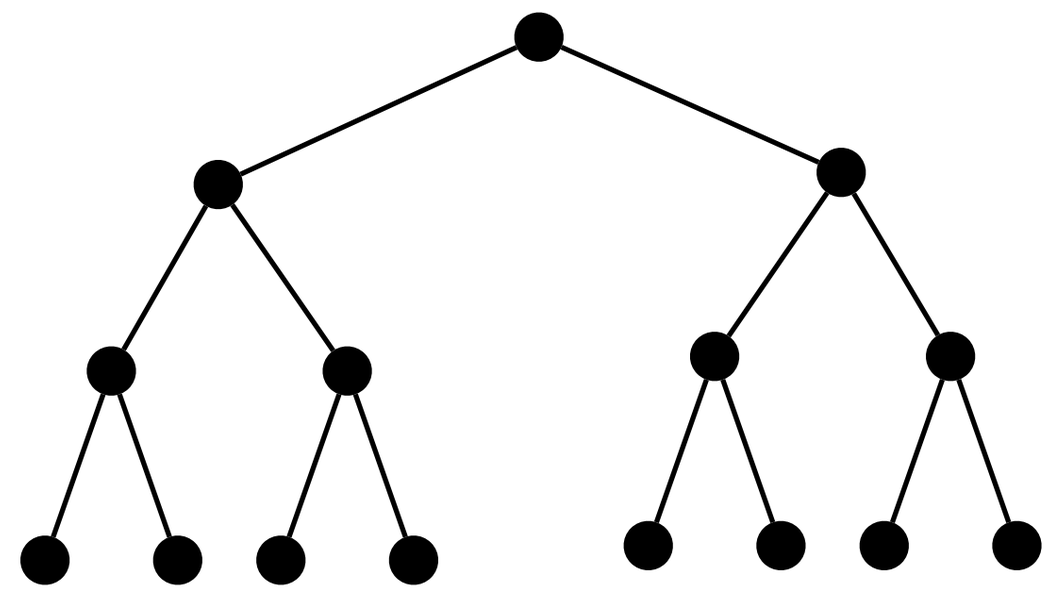
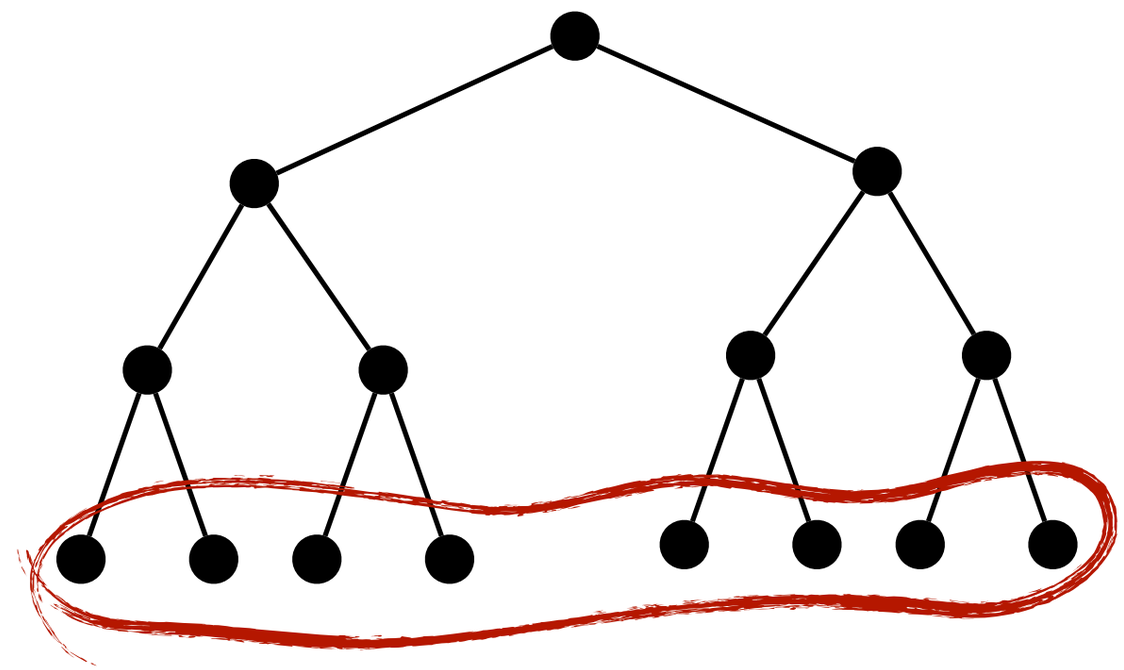


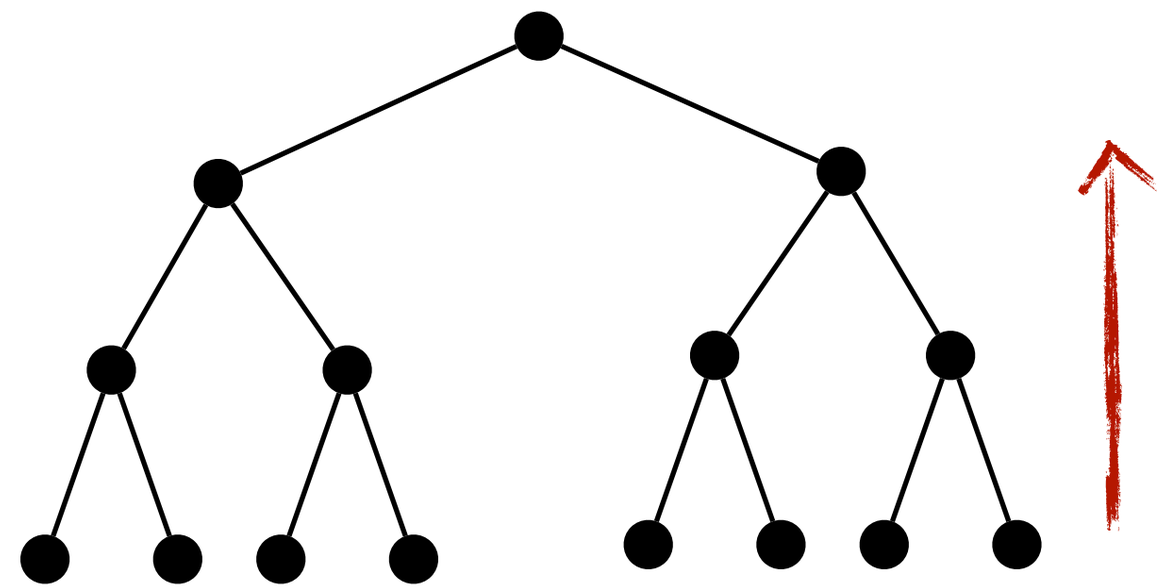
The Standard Approach

- Generate all assignments
- Evaluate the assignments

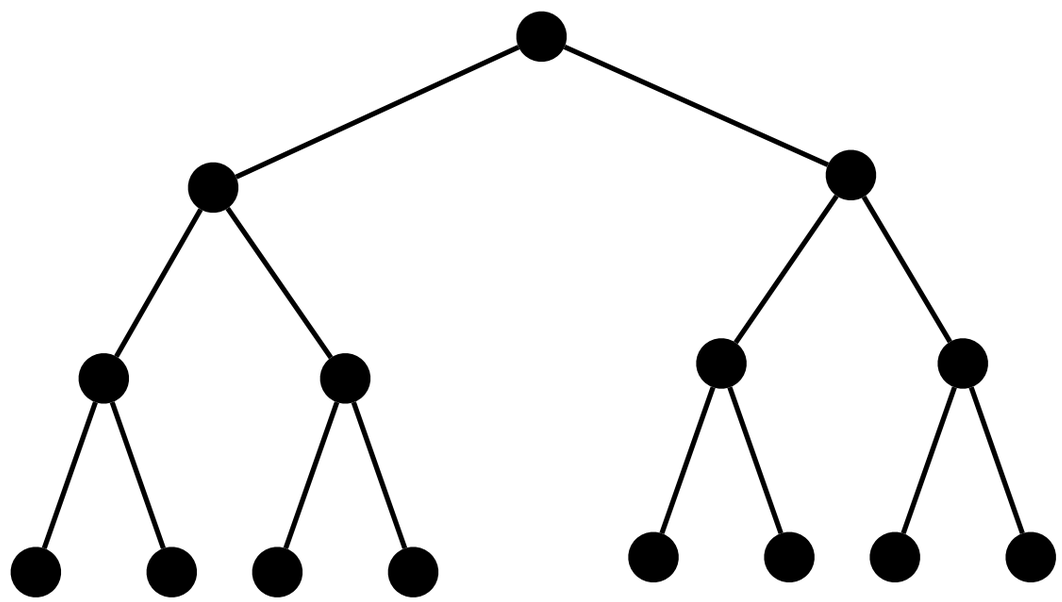


The Standard Approach

- Generate all assignments
- Evaluate the assignments
- Combine the results according to the quantifiers

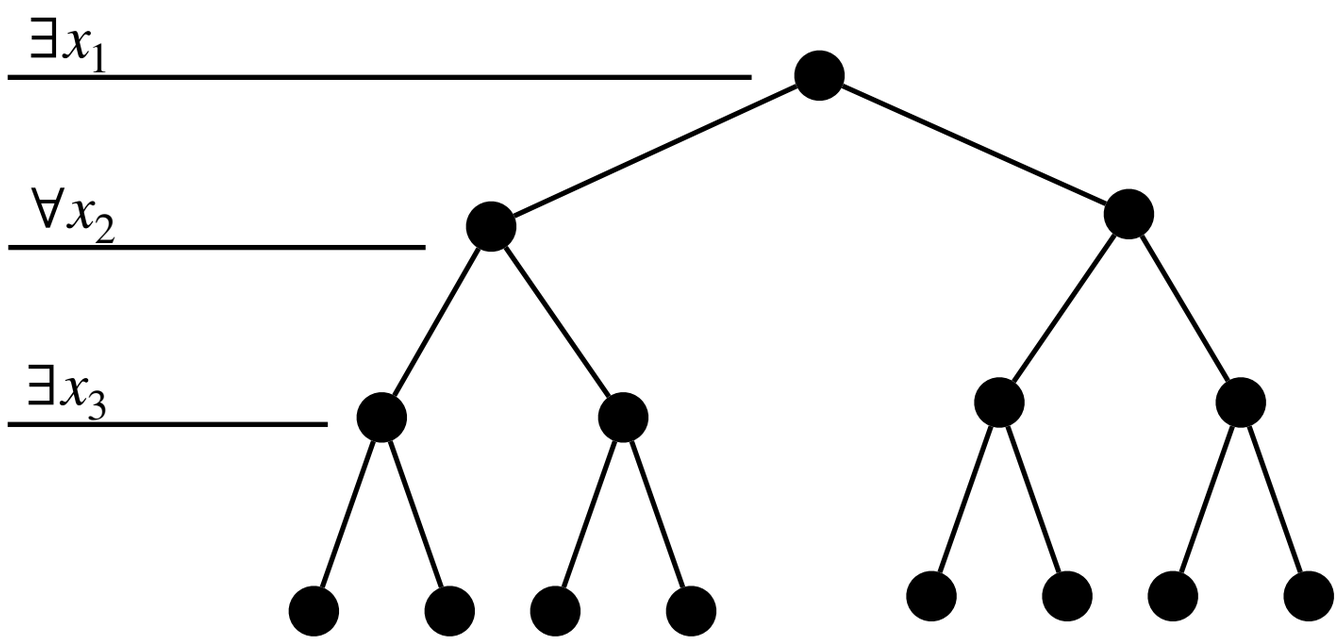


Usual Solutions



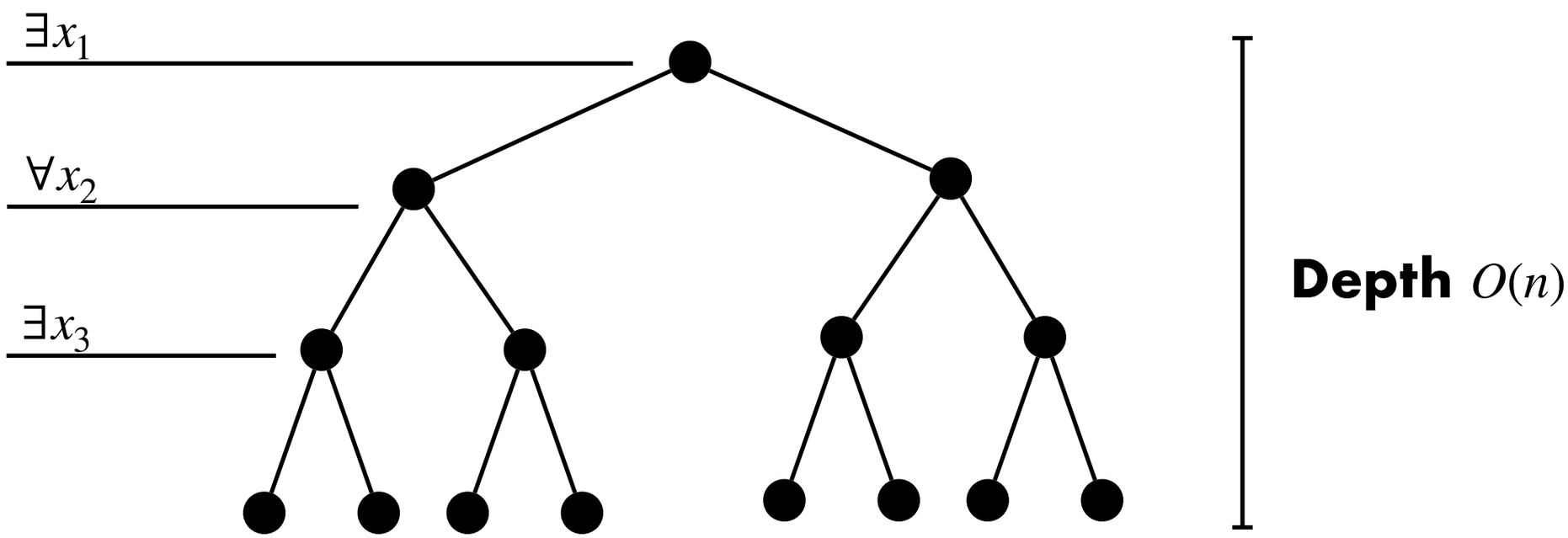
Membrane structure

Usual Solutions



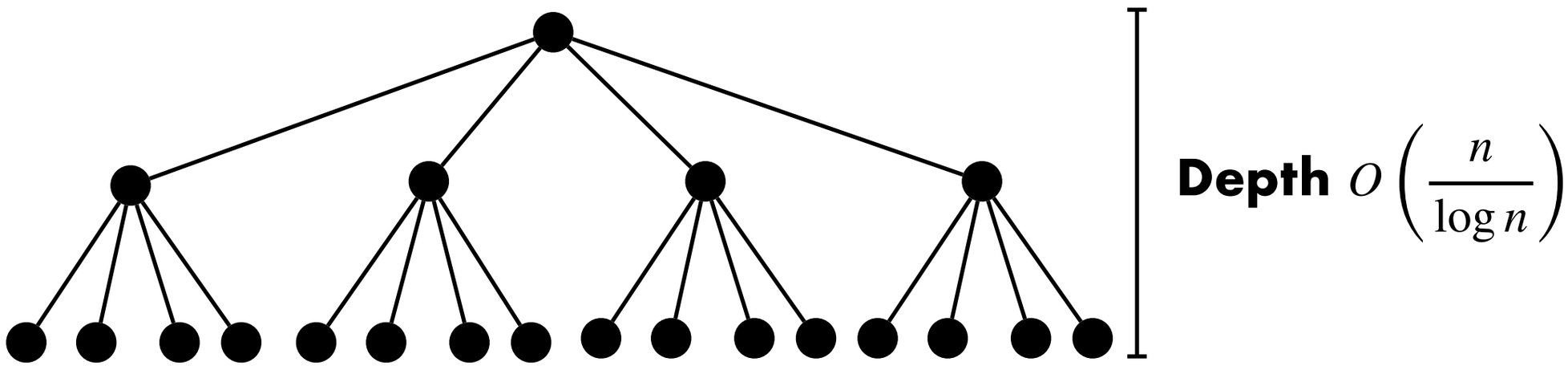
Membrane structure

Usual Solutions



Membrane structure

New Approach



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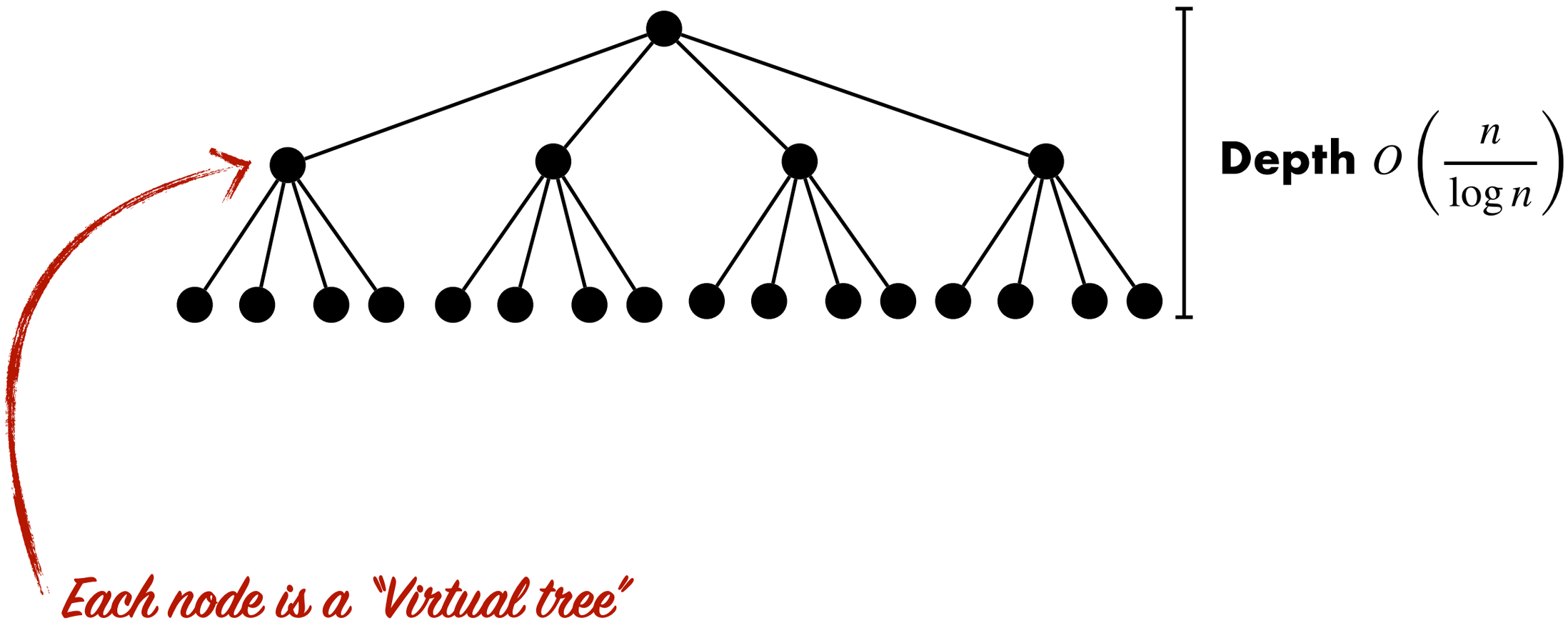
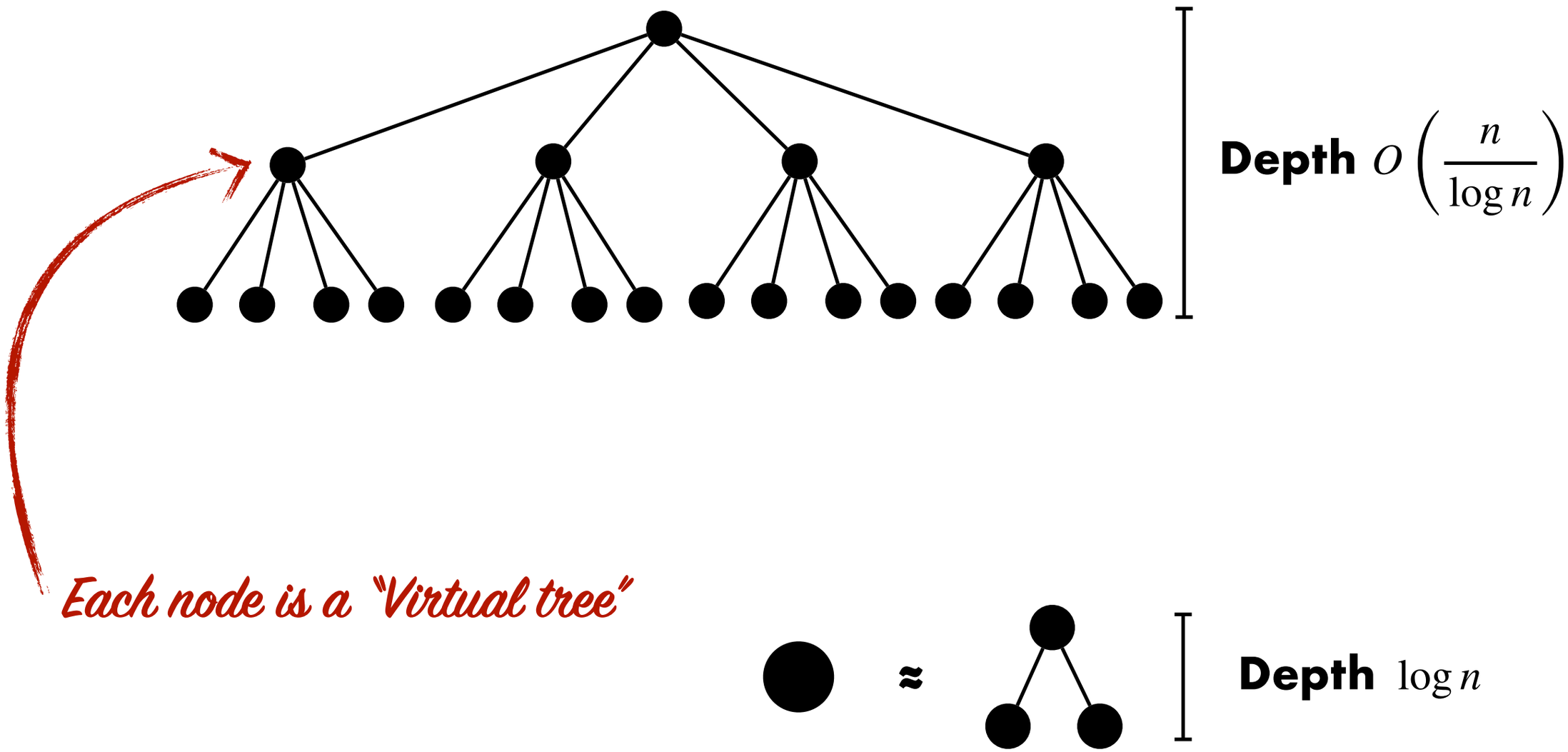


New Approach



Steps of the algorithm

Steps of the algorithm

- Generation of the assignments

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Steps of the algorithm

- Generation of the assignments *Simple*
- Evaluation of the assignments

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- Evaluation of the assignments *Works as usual*
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This is the complex part

Enhanced Charges

$$\Psi = \{-,0,+\}$$

Three “classical” charges

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Three "classical" charges

$$\Psi = \{\alpha_1, \alpha_2, \ldots, \alpha_{p(n)}\}$$

Polynomially many charges

Enhanced Charges

$$\Psi = \{-,0,+\}$$

Three "classical" charges

$$\Psi = \{\alpha_1, \alpha_2, \ldots, \alpha_{p(n)}\}$$

Polynomially many charges

Simulable with 3 charges with polynomial slowdown

Blocks of Quantifiers

$$\exists x_1 \forall x_2 \exists x_3 \ \forall x_4 \exists x_5 \forall x_6 \ \exists x_7 \forall x_8 \exists x_9 \ \forall x_{10} \exists x_{11} \forall x_{12} \ \varphi(x_1, \ldots, x_{12})$$

Blocks of Quantifiers

$$\boxed{\exists x_1 \forall x_2 \exists x_3}\boxed{\forall x_4 \exists x_5 \forall x_6}\boxed{\exists x_7 \forall x_8 \exists x_9}\boxed{\forall x_{10} \exists x_{11} \forall x_{12}}\varphi(x_1, \ldots, x_{12})$$

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$\frac{n}{\log n}$ **blocks of** $\log n$ **quantifiers**

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One block ≈ a level in the membrane structure

Blocks of Quantifiers

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Skin

$\frac{n}{\log n}$ **blocks of** $\log n$ **quantifiers**

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Blocks of Quantifiers

Depth 1

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Blocks of Quantifiers

Depth 1

Depth 2

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Blocks of Quantifiers

Depth 1 Depth 2 Depth 3

$$\boxed{\exists x_1 \forall x_2 \exists x_3}\boxed{\forall x_4 \exists x_5 \forall x_6}\boxed{\exists x_7 \forall x_8 \exists x_9}\boxed{\forall x_{10} \exists x_{11} \forall x_{12}}\varphi(x_1, \ldots, x_{12})$$

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Skin

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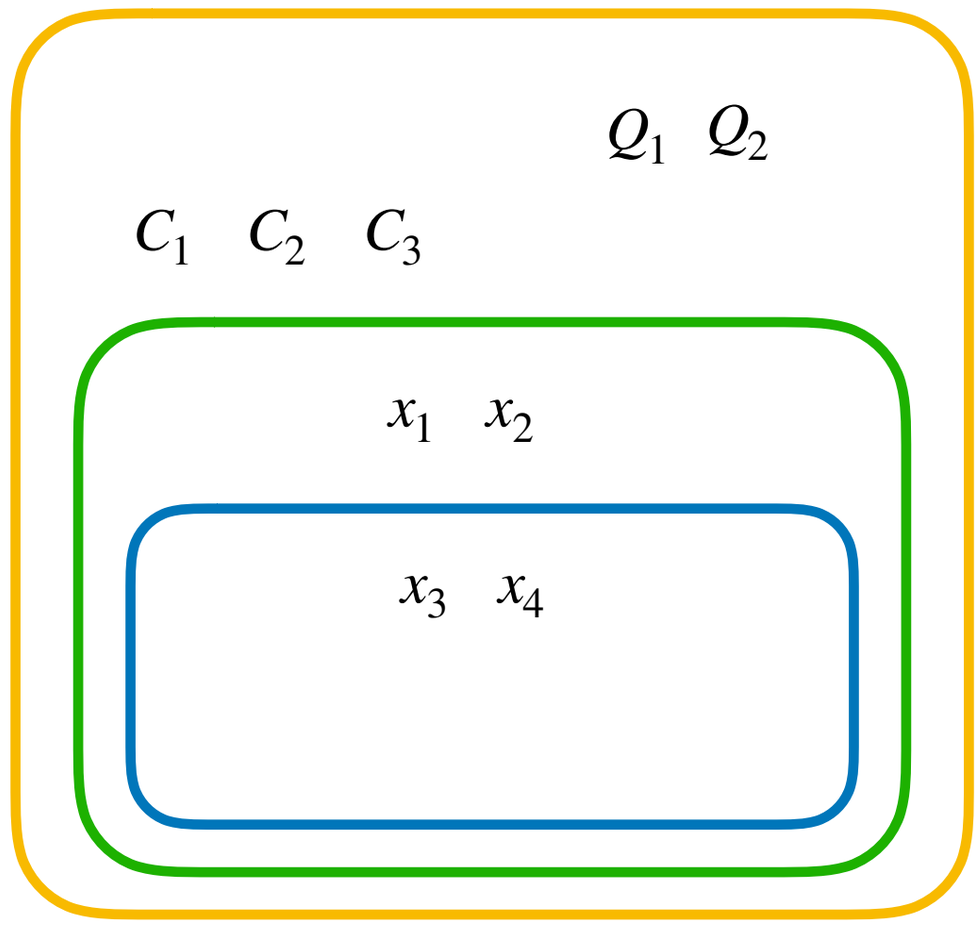
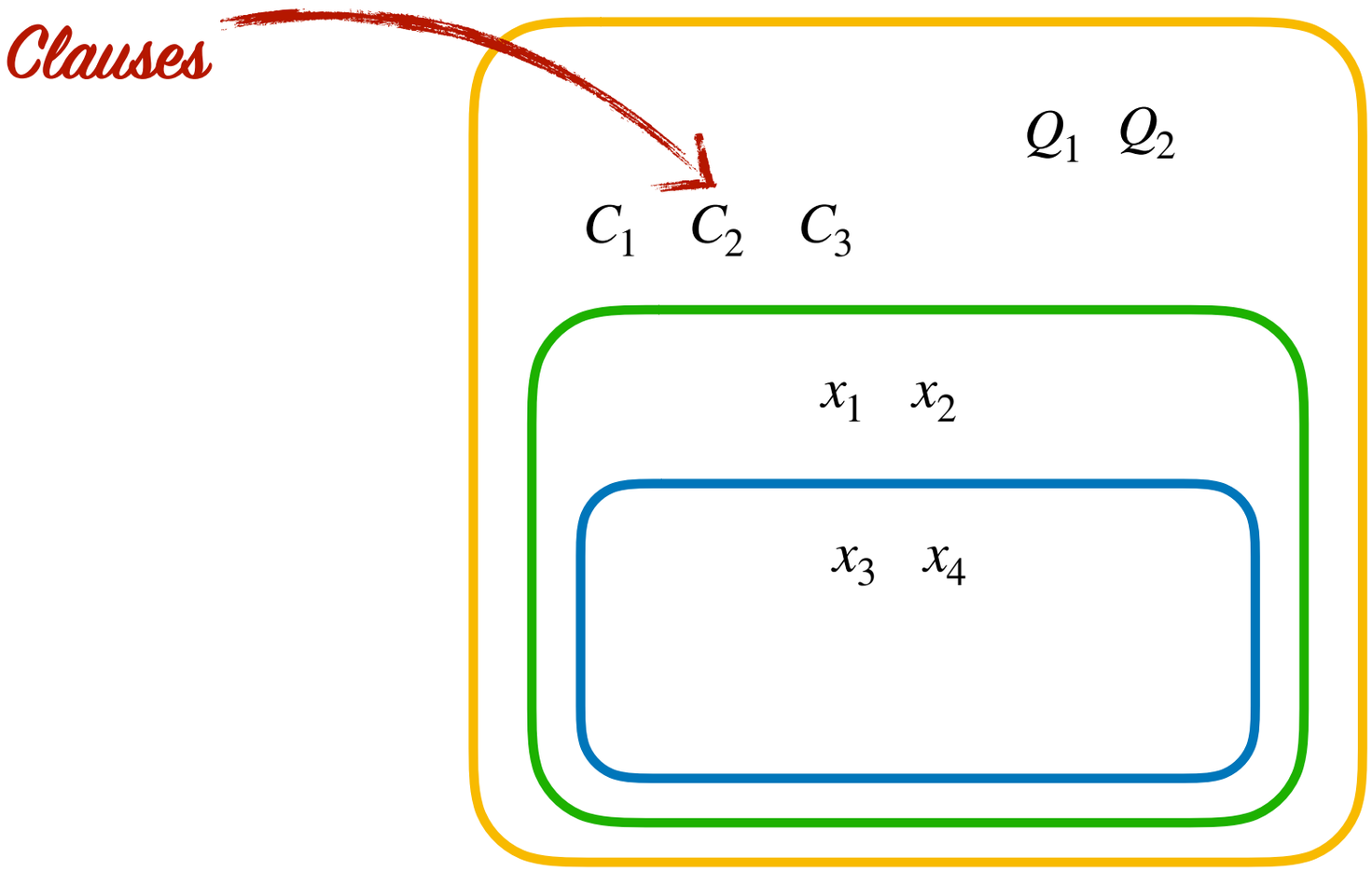
Elementary Membranes

One block ≈ a level in the membrane structure

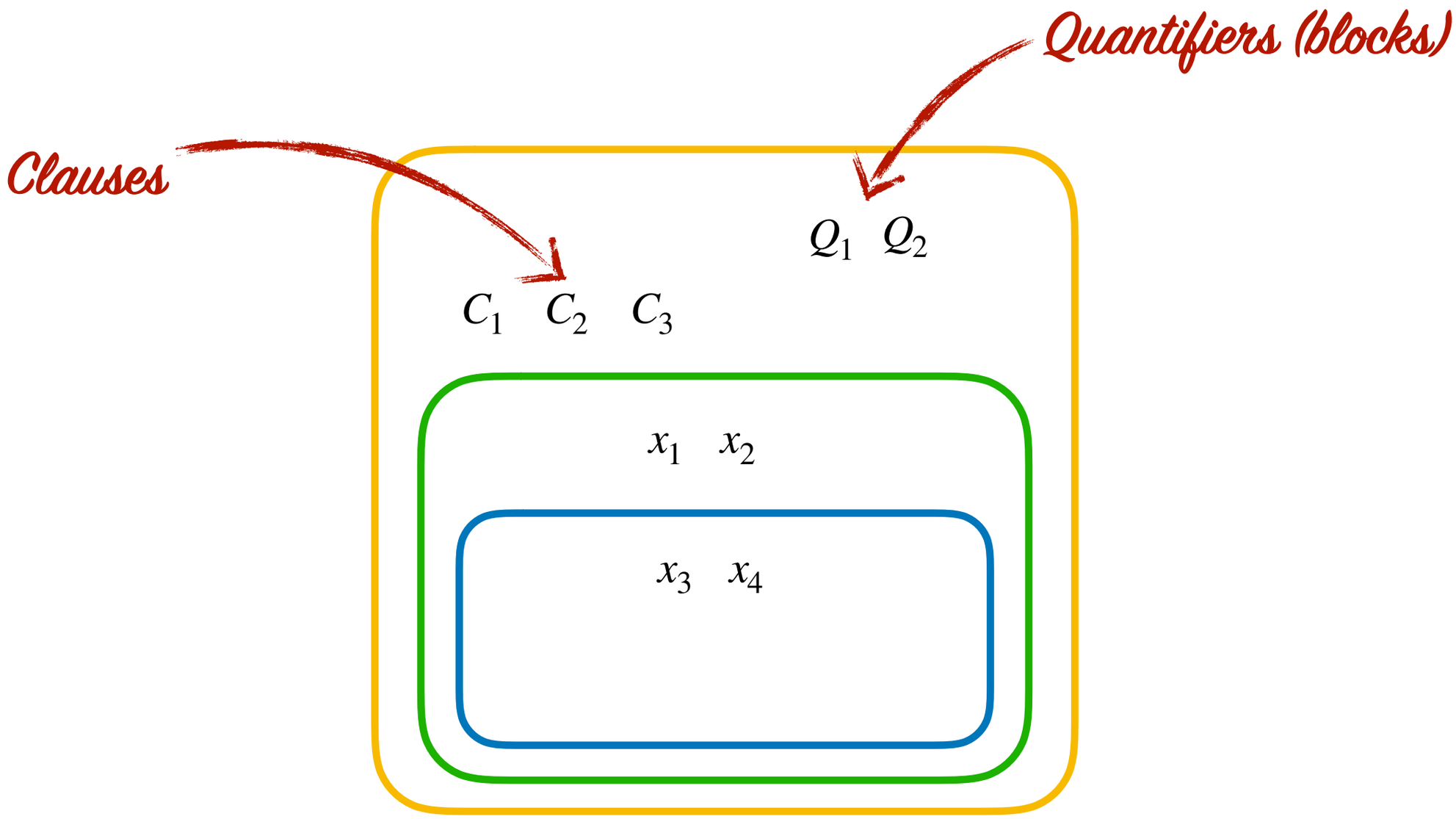
Initial Configuration



Initial Configuration



Initial Configuration



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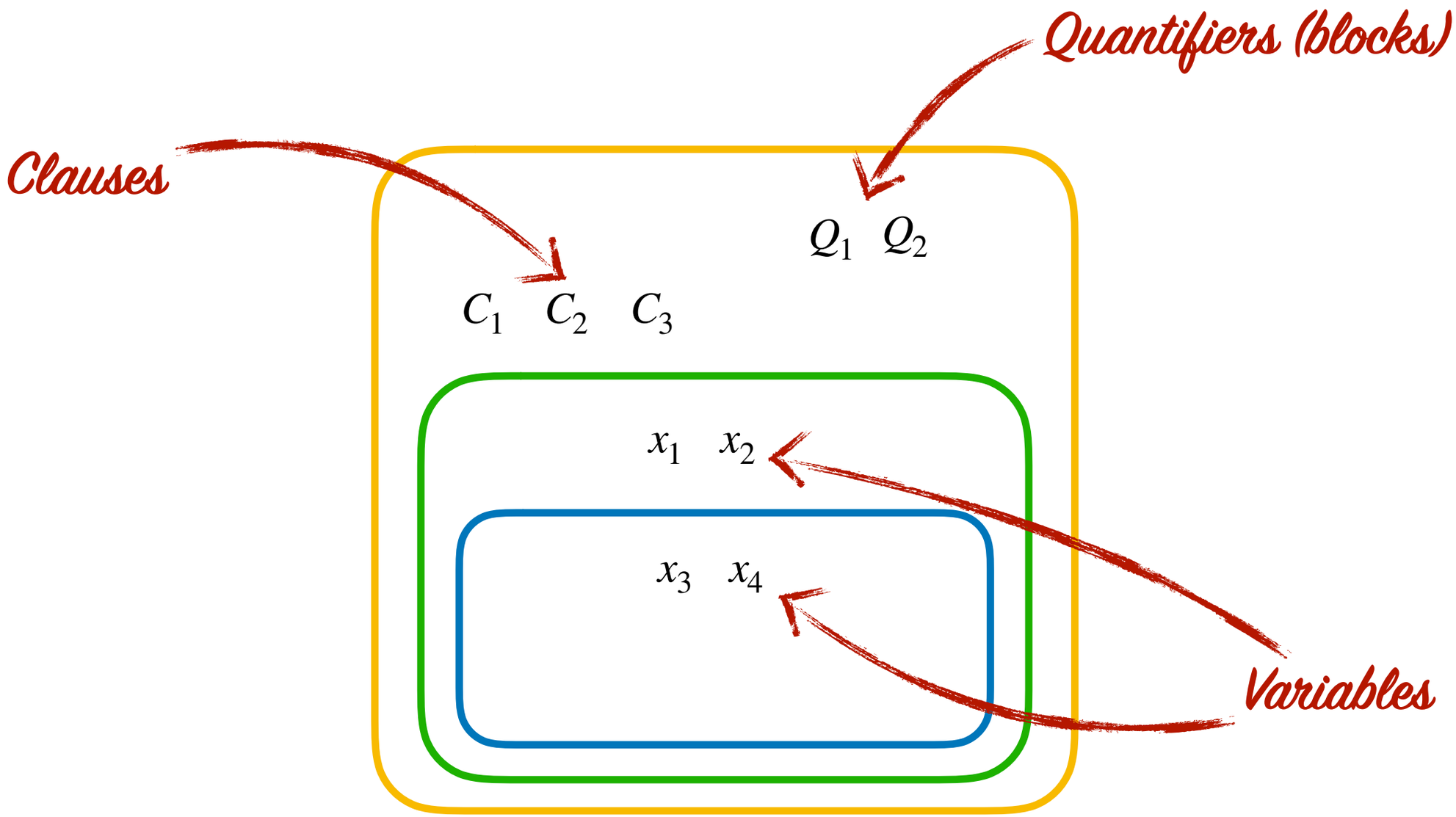
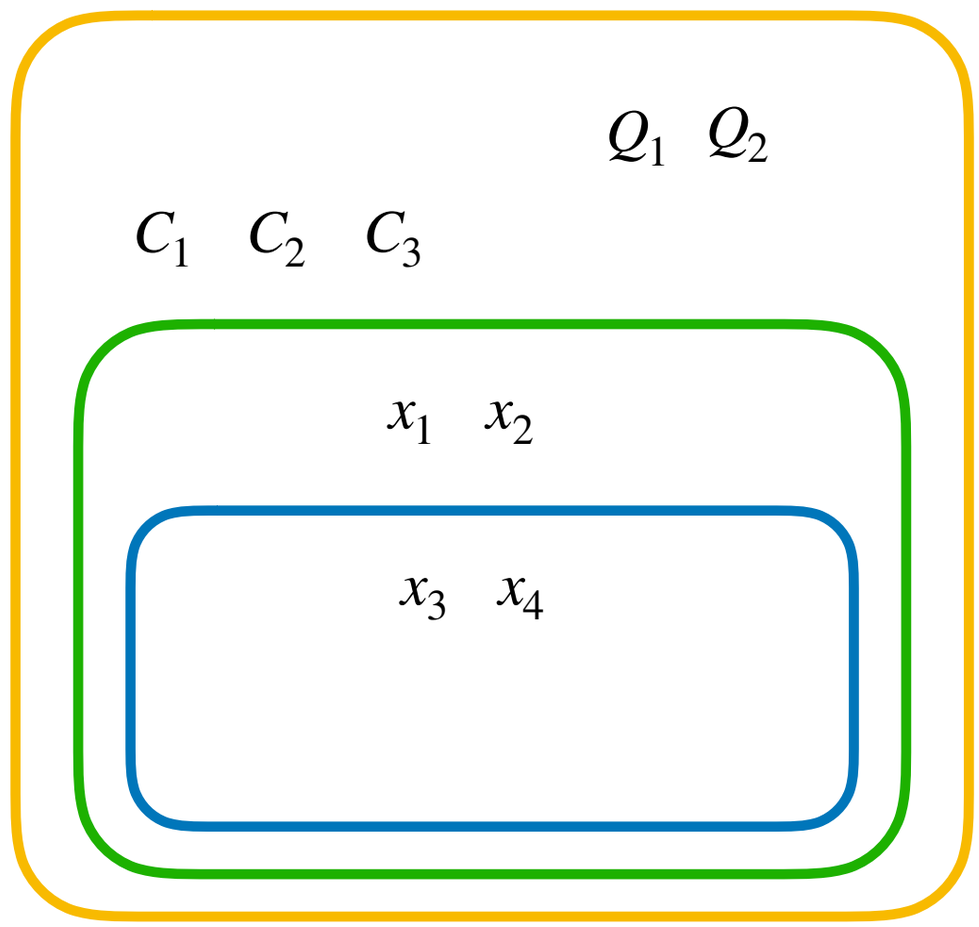
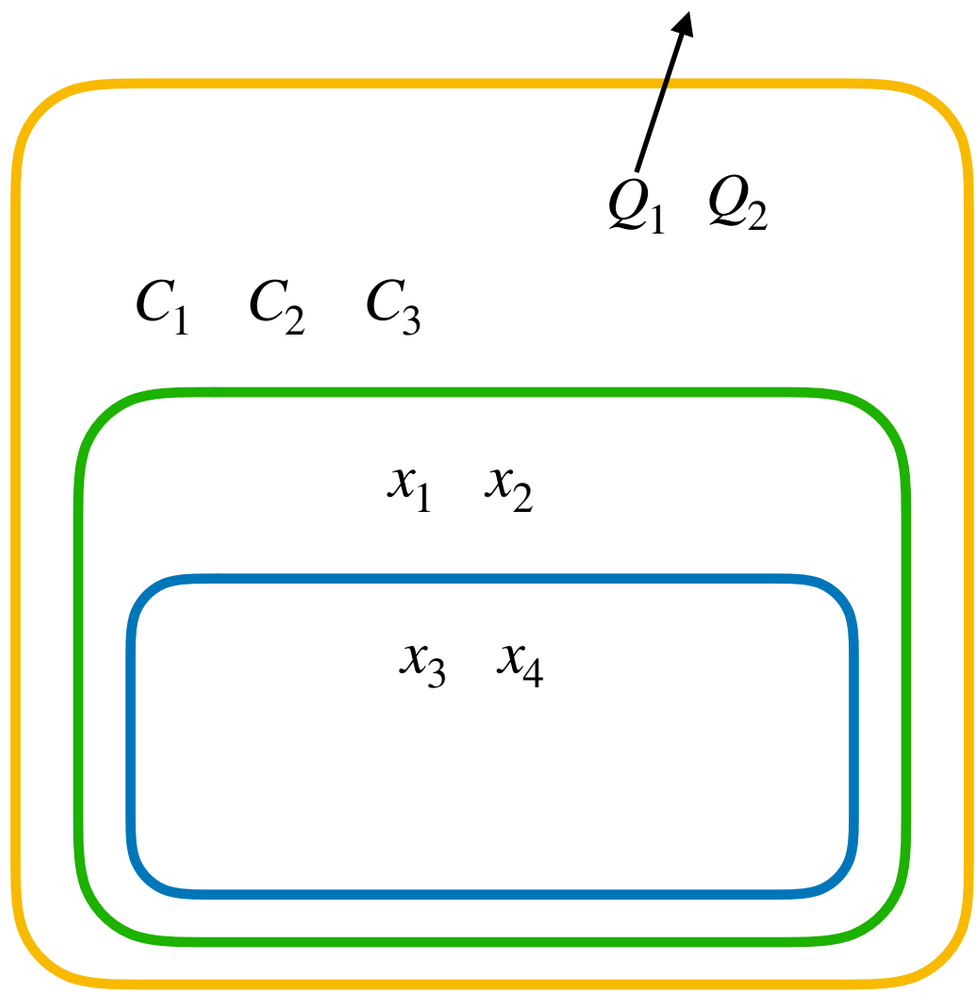


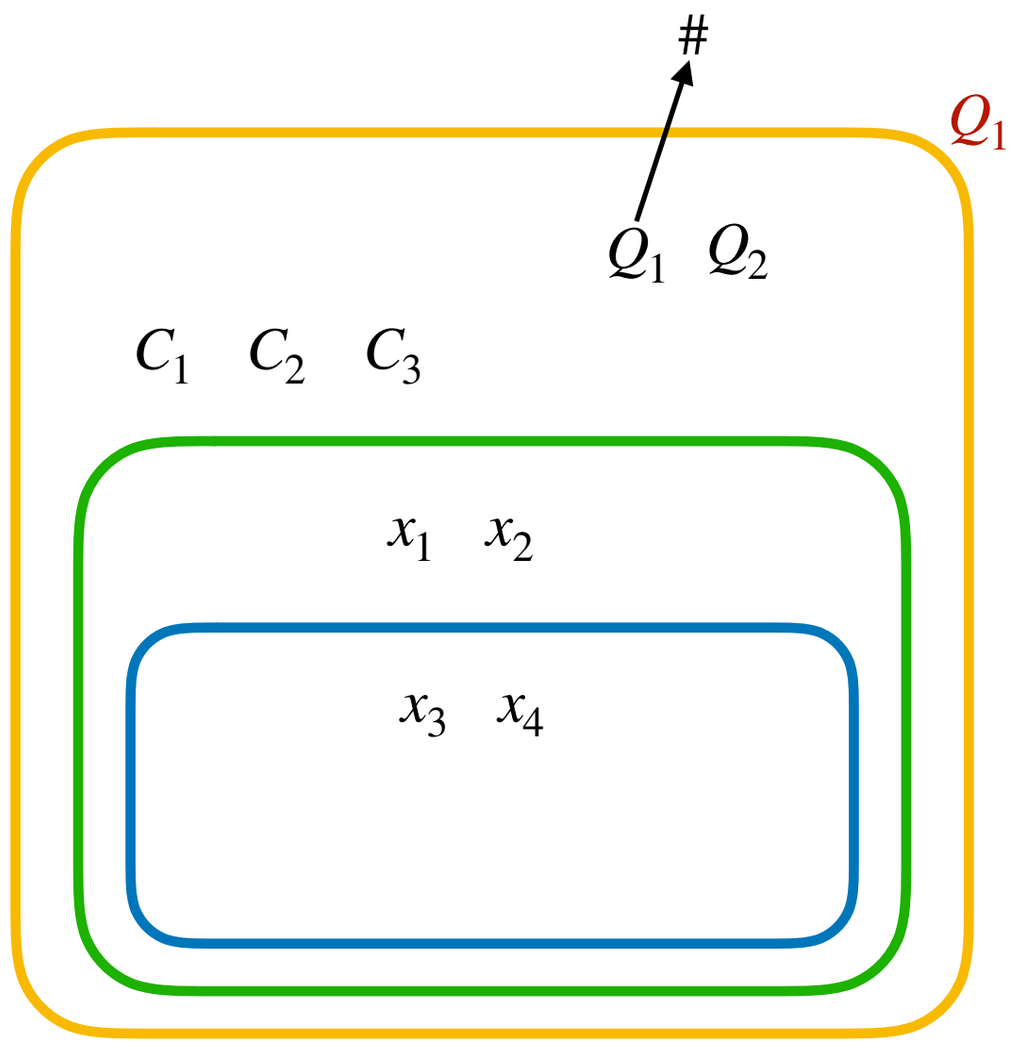
Quantifiers Propagation



Quantifiers Propagation



Quantifiers Propagation



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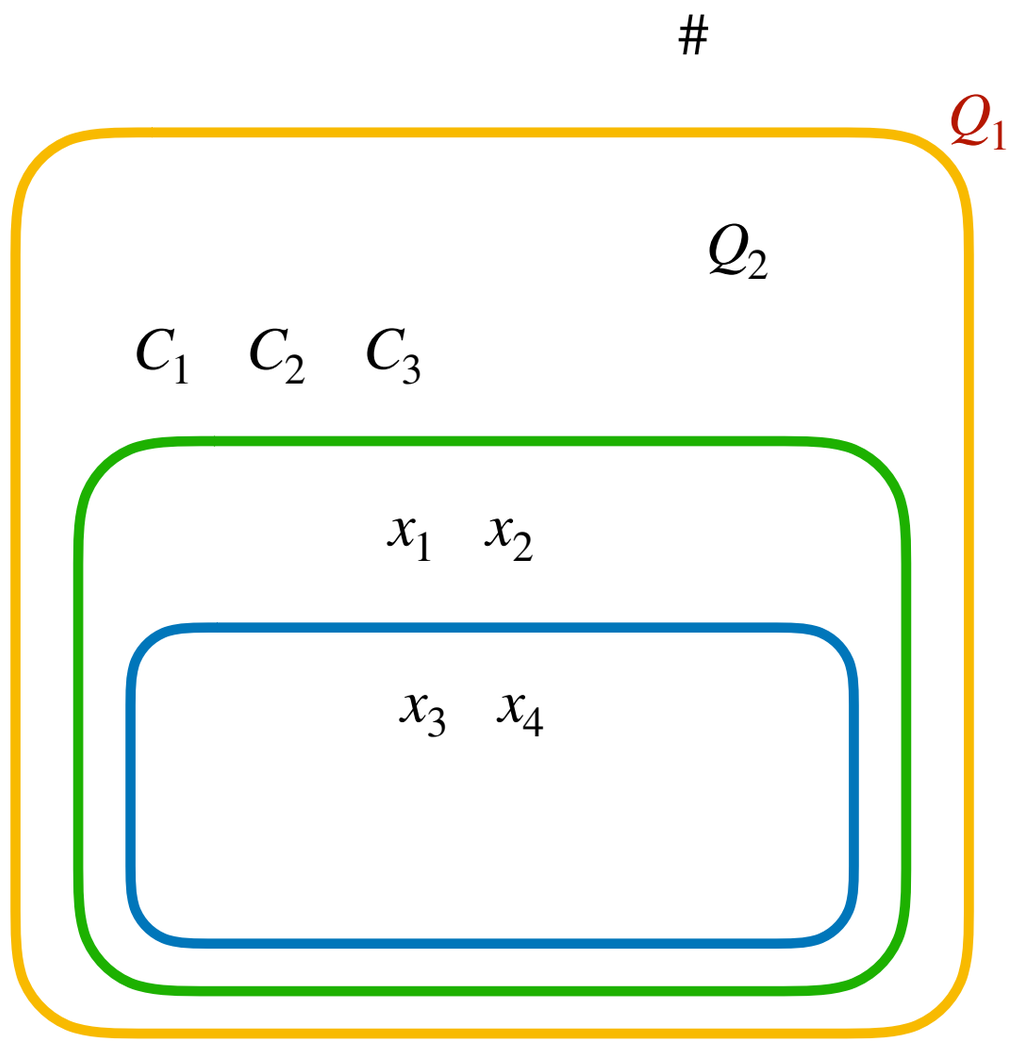
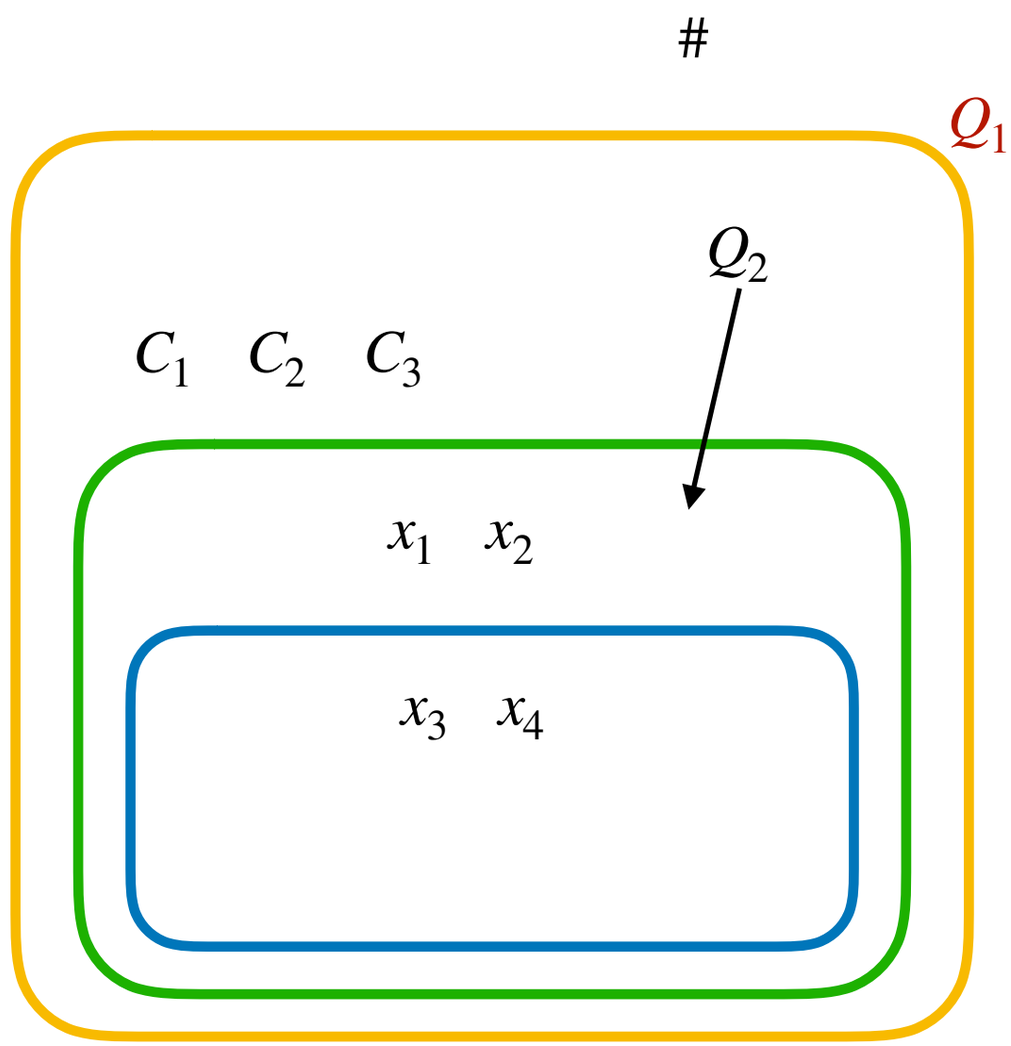


Quantifiers Propagation



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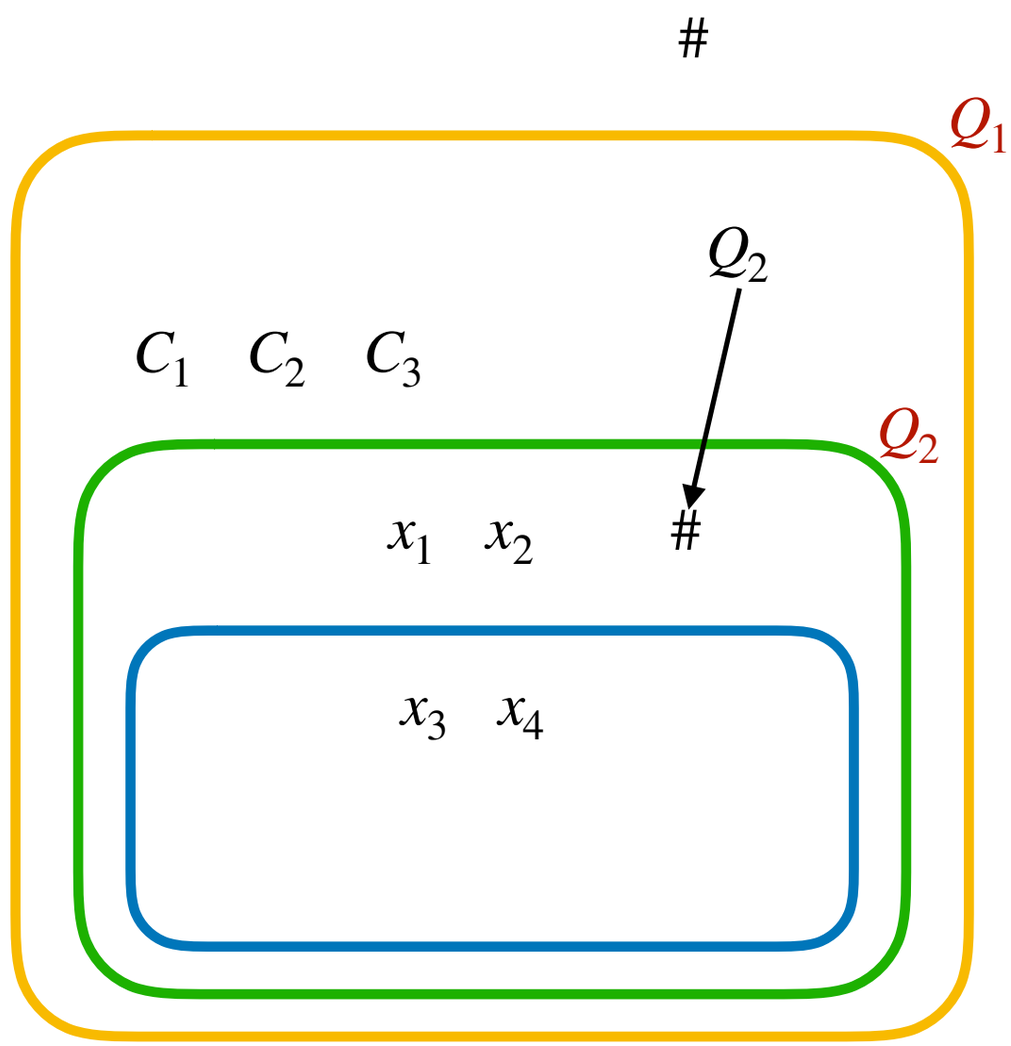

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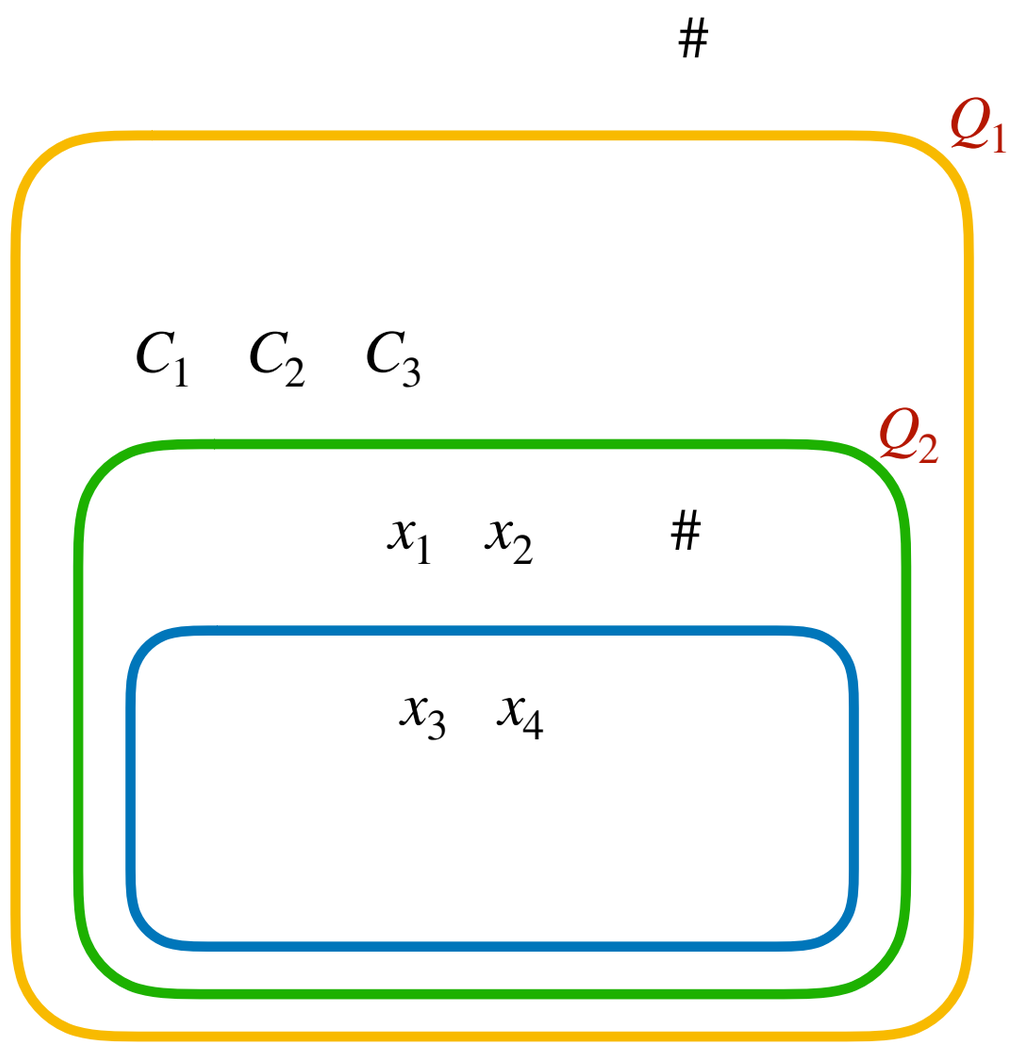

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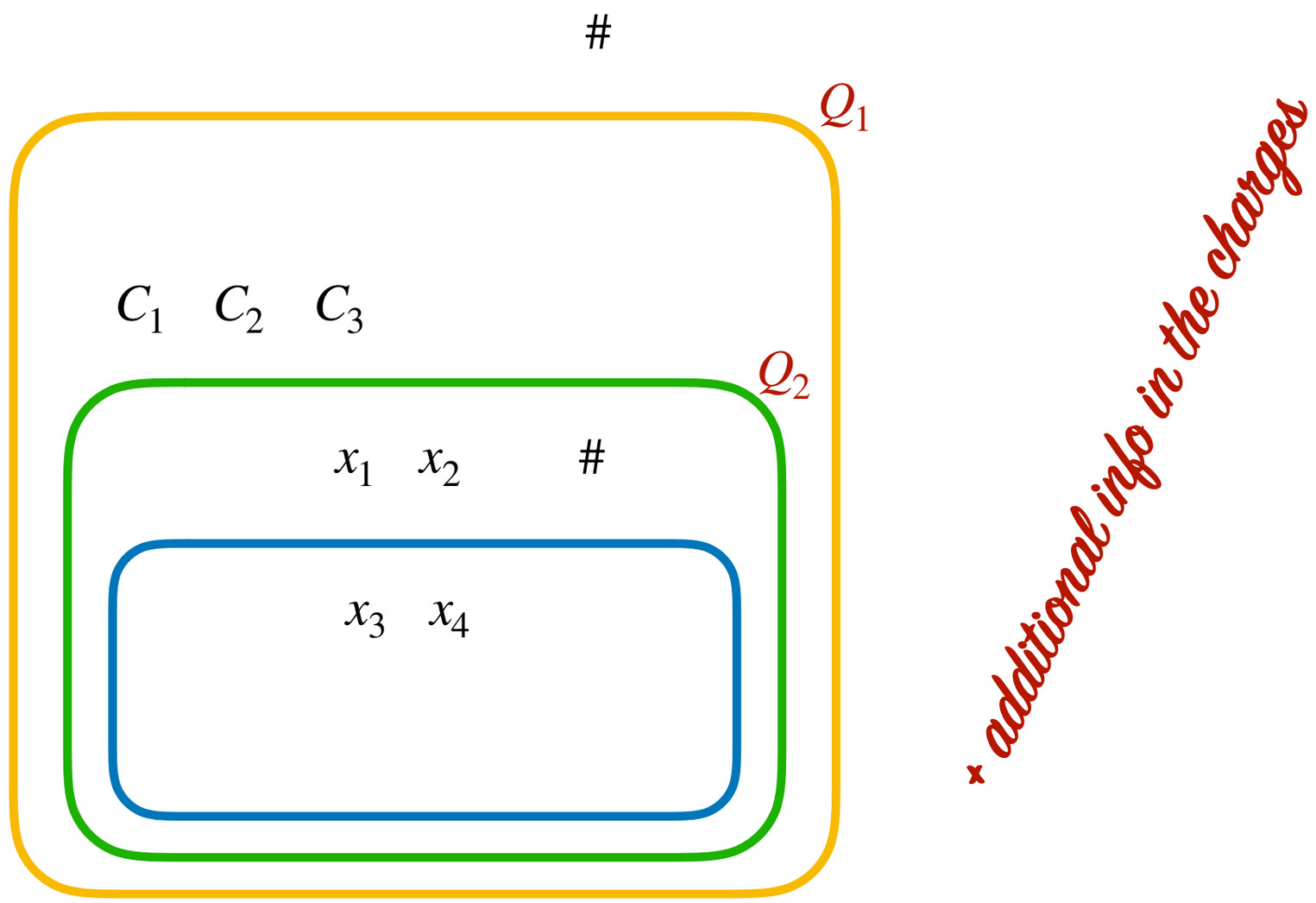
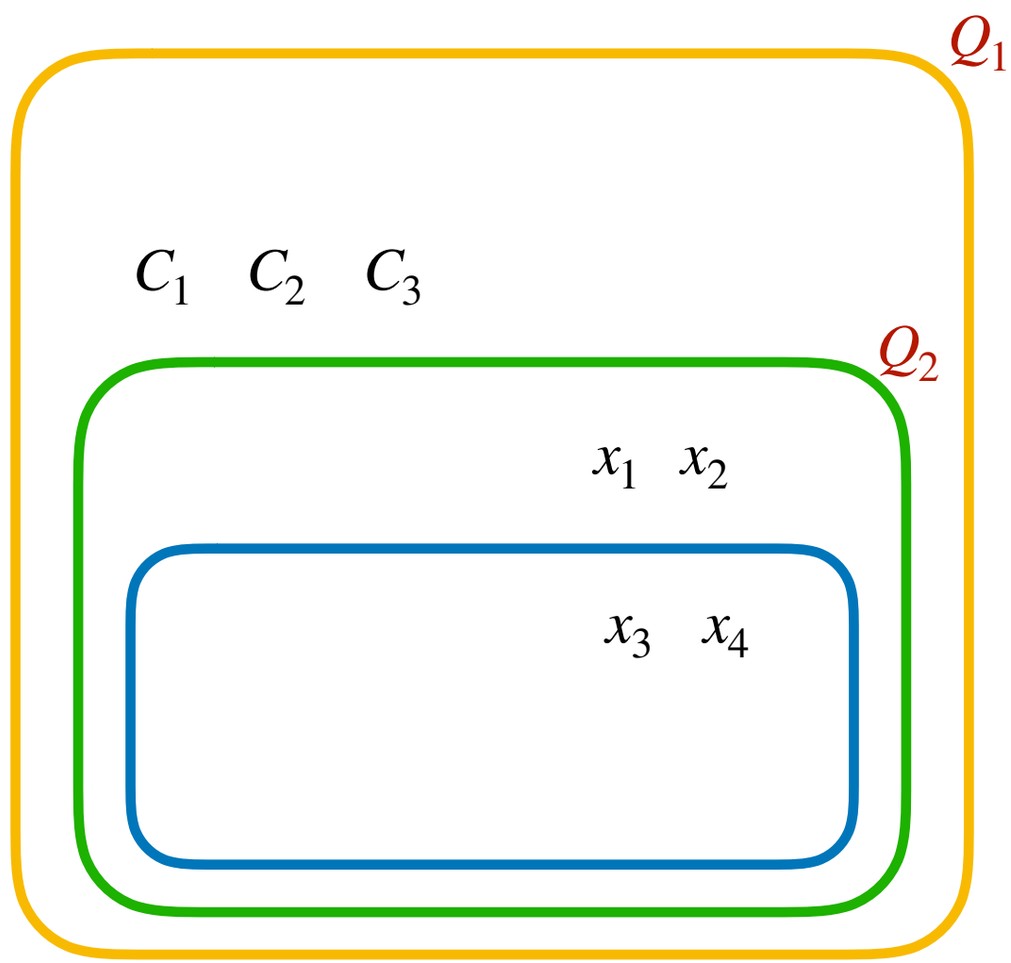


Sending Clauses Down



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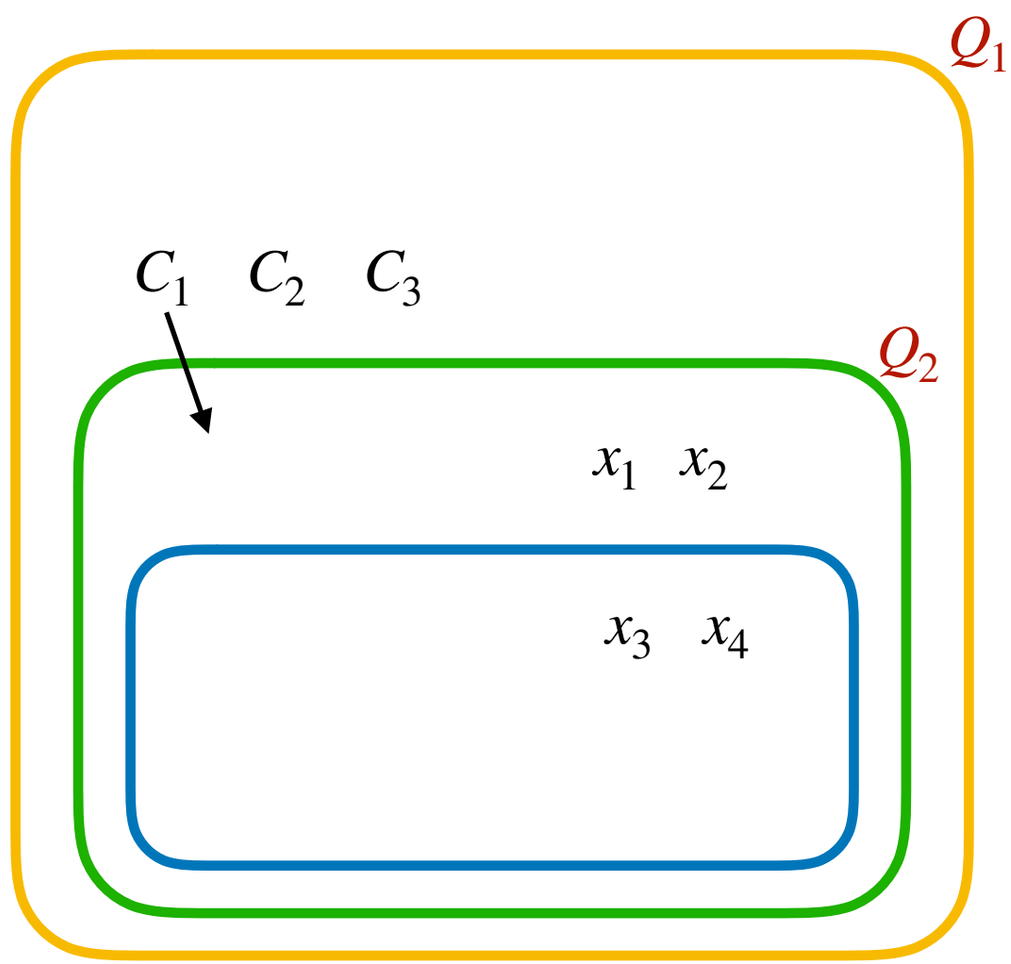

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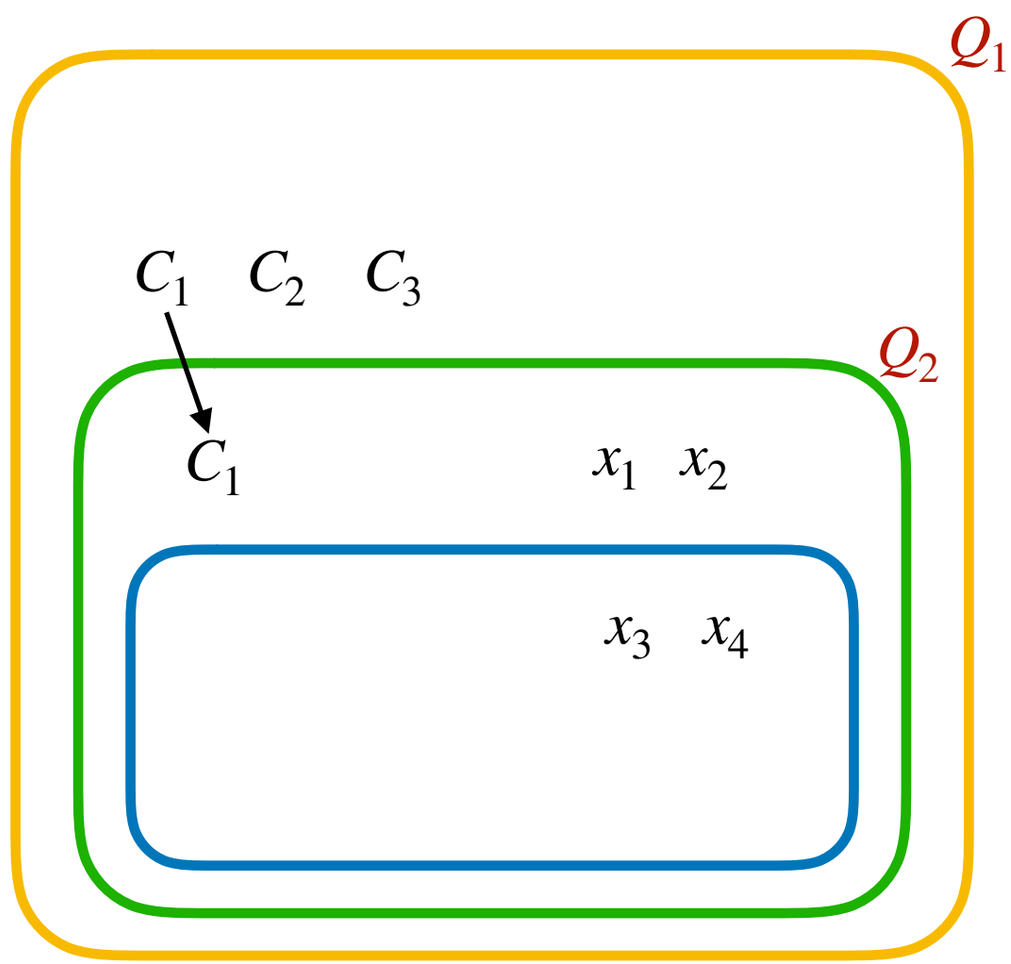

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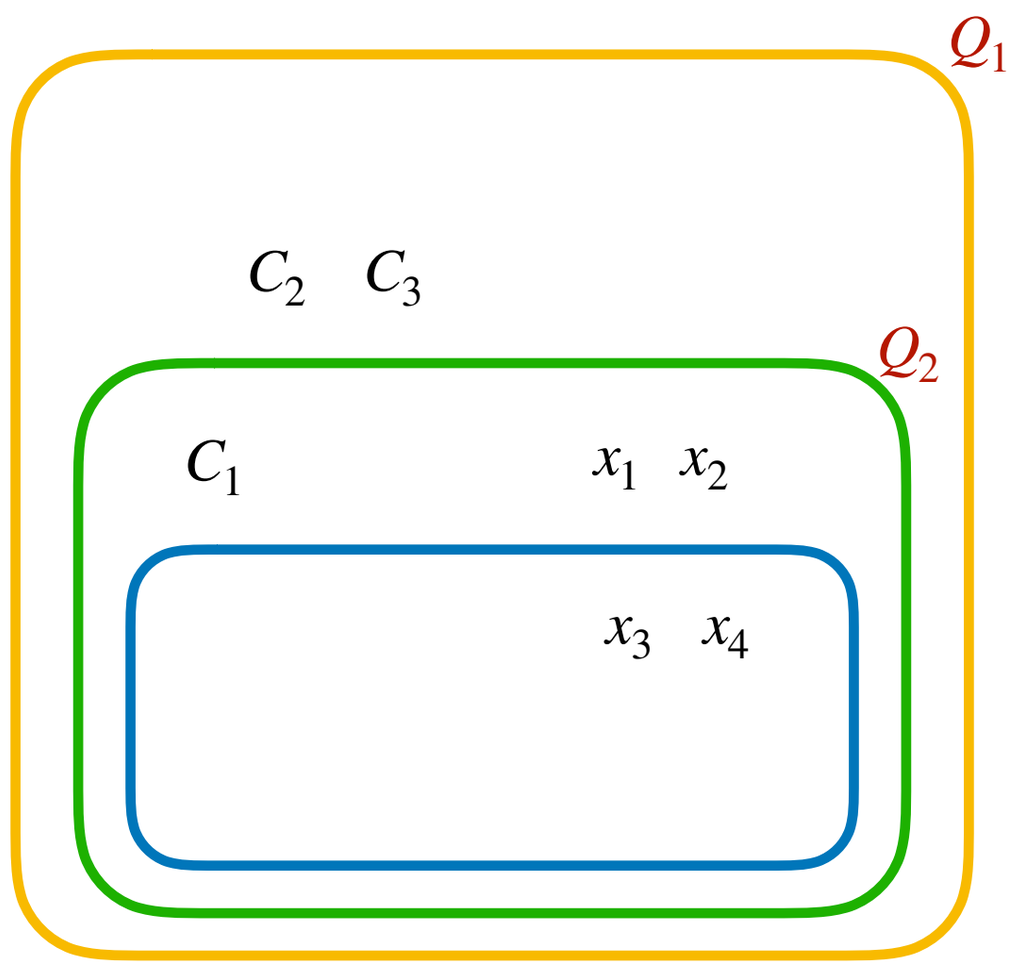

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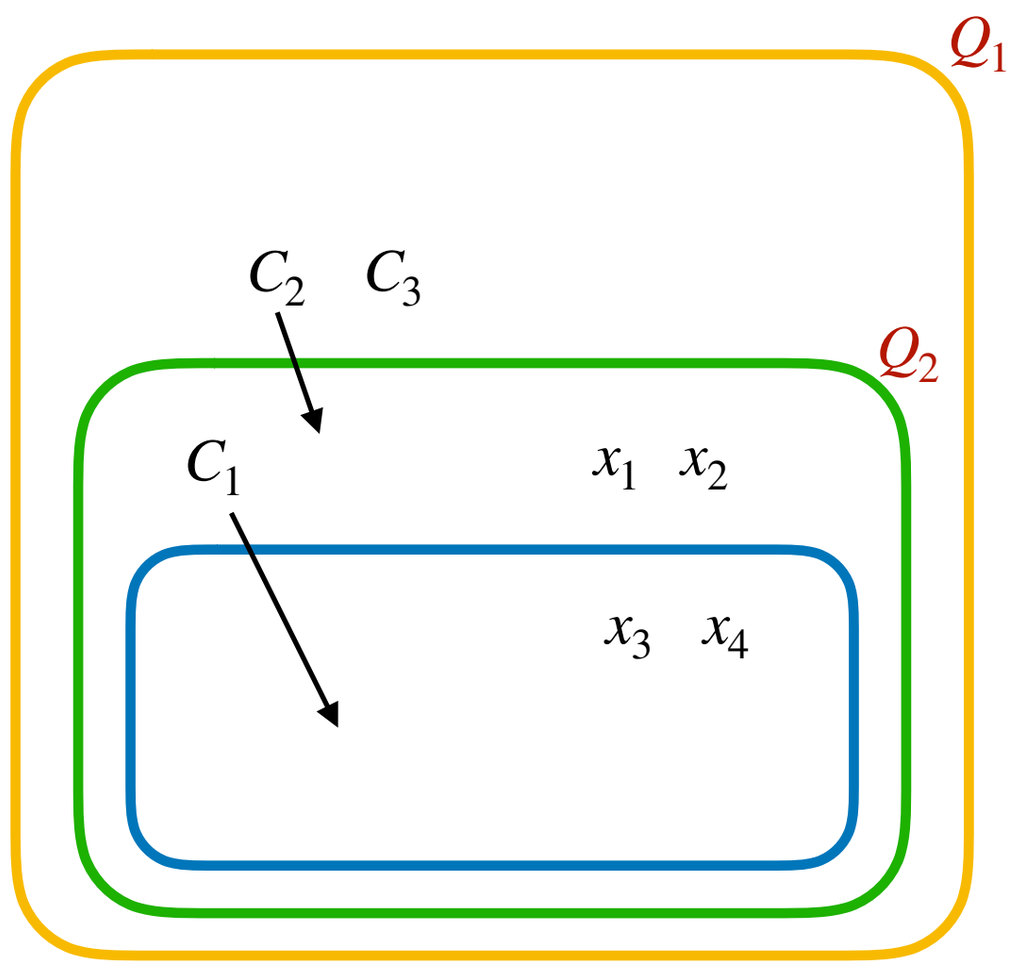

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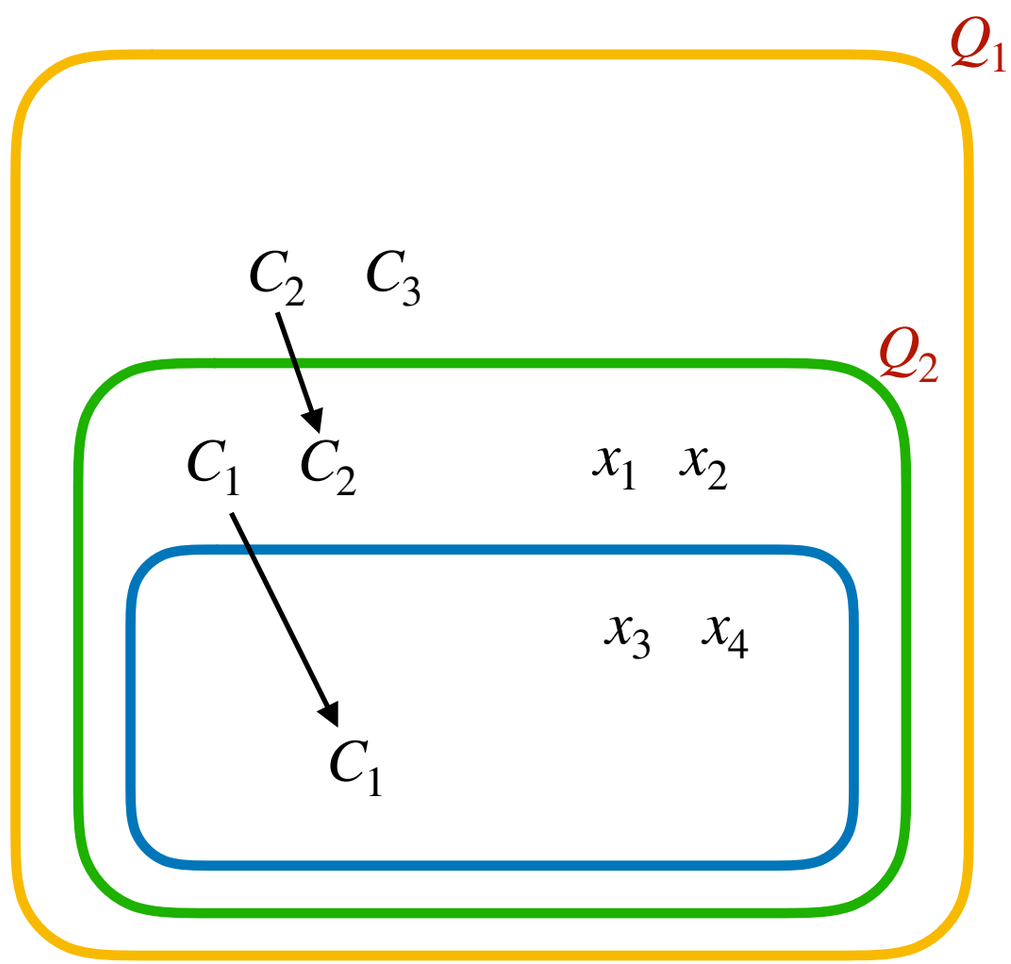

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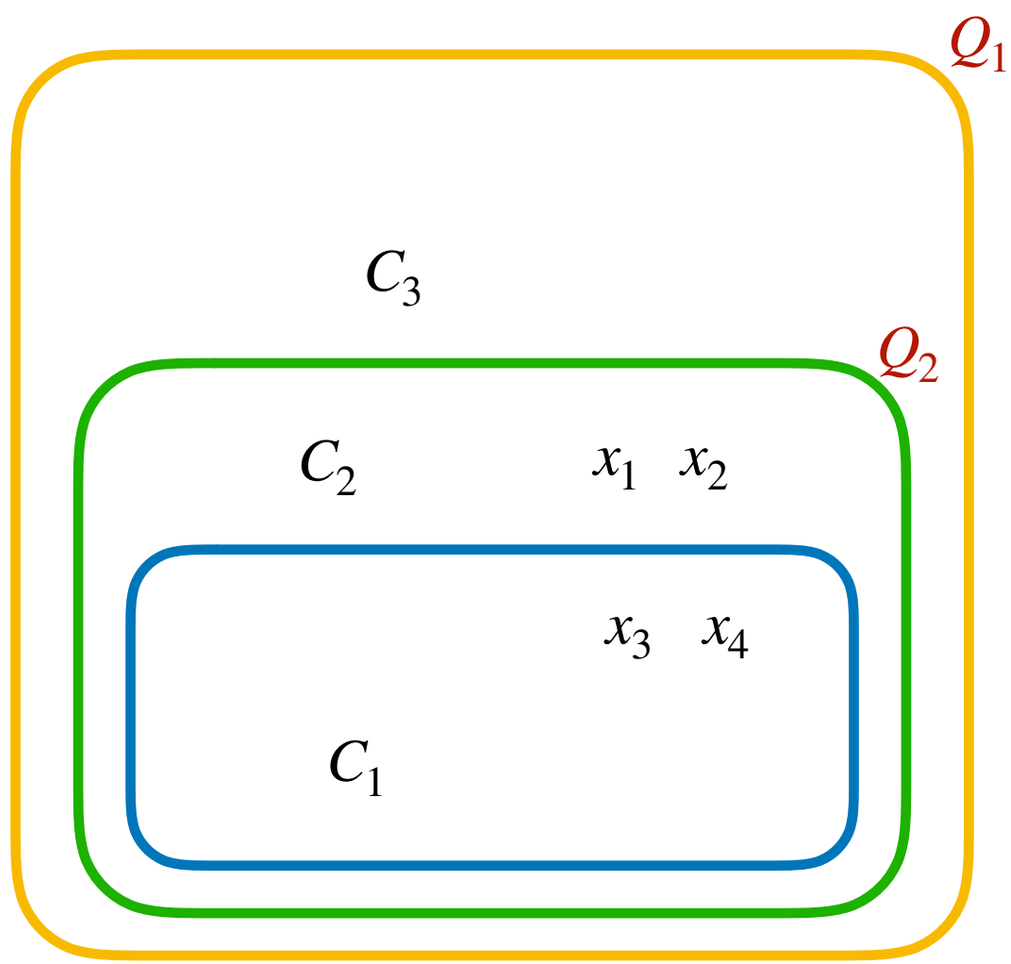

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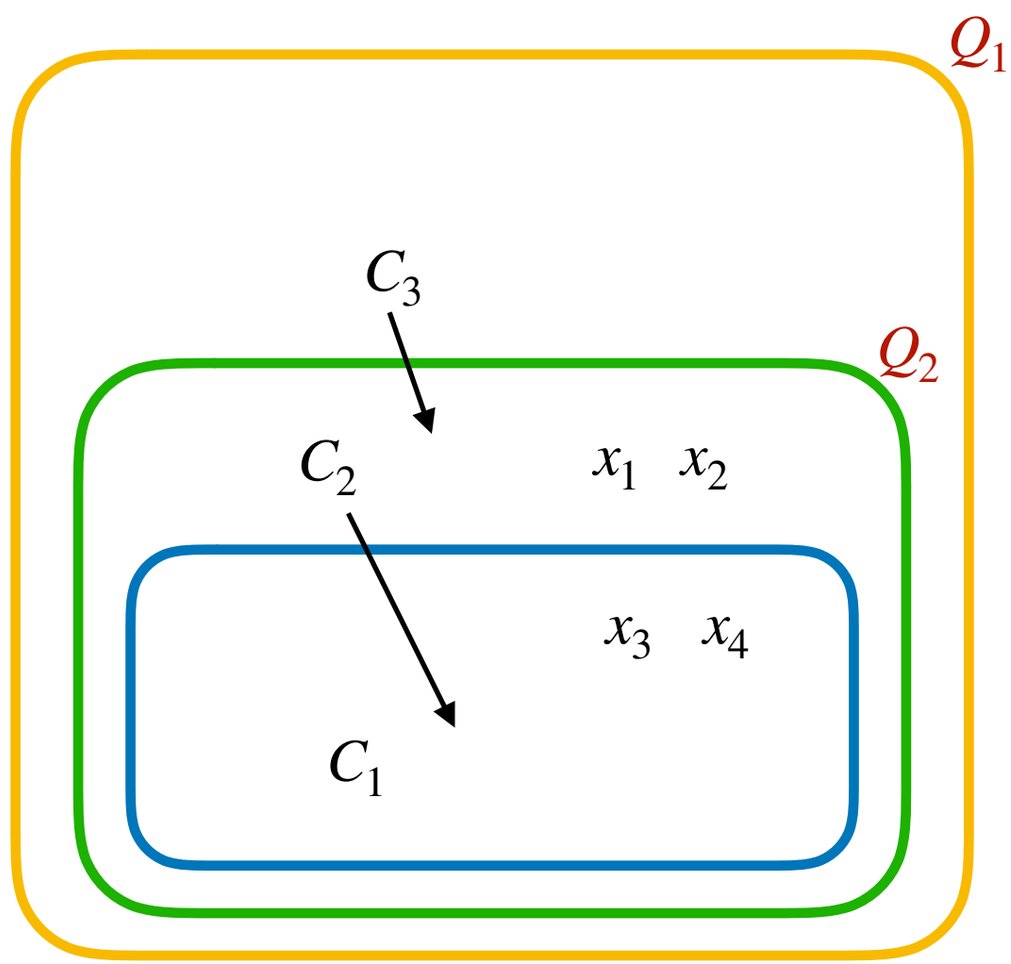

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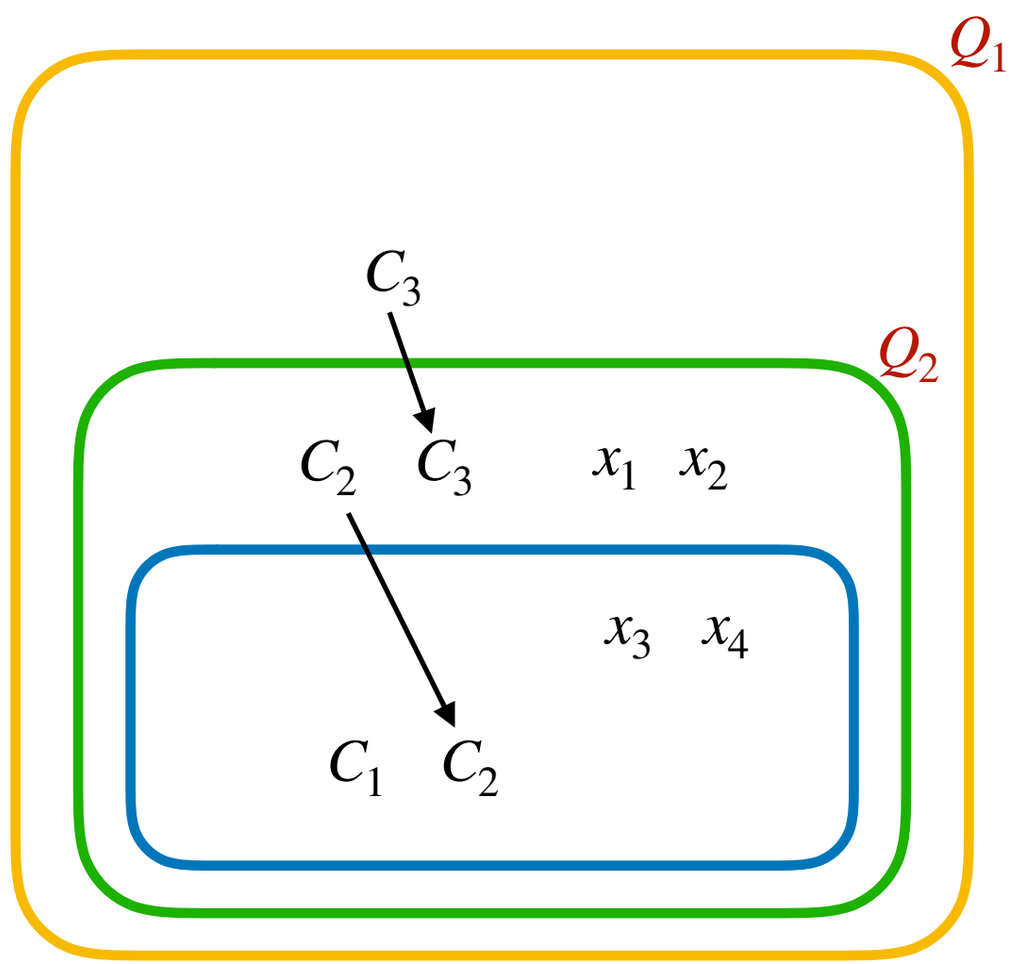

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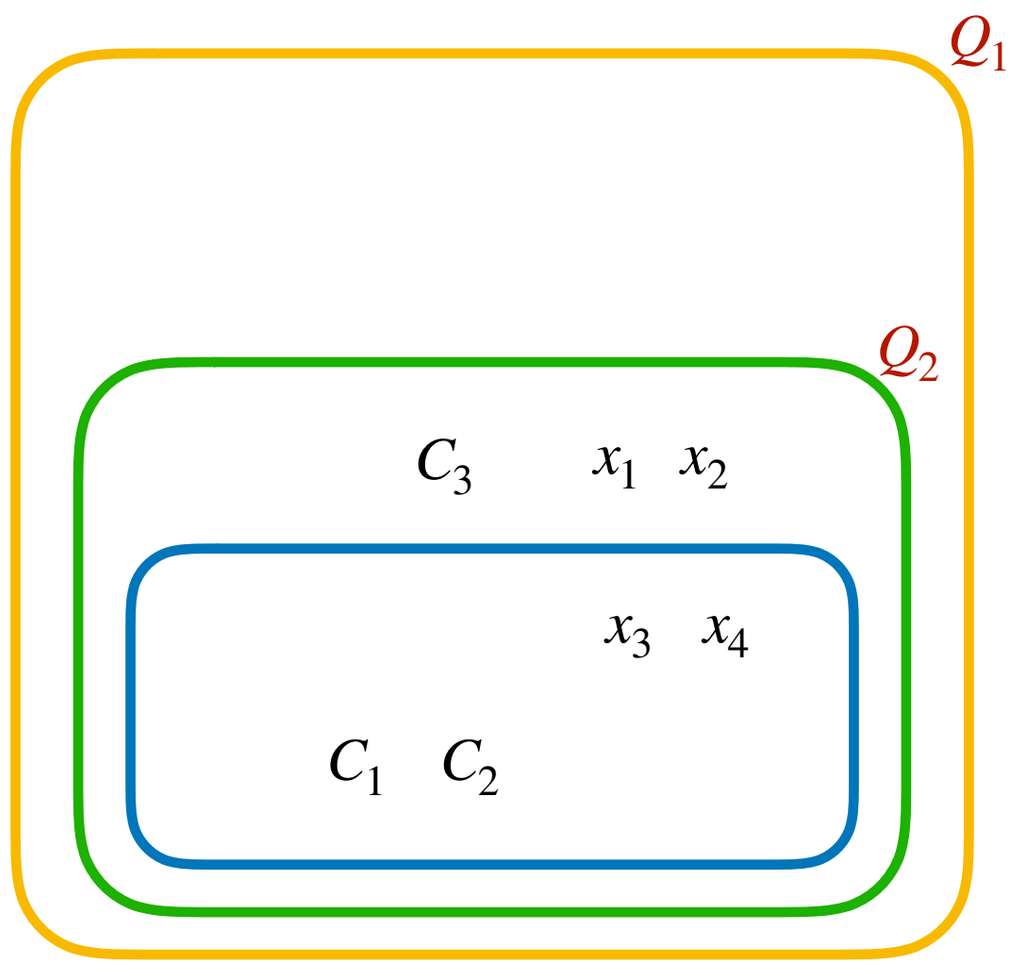

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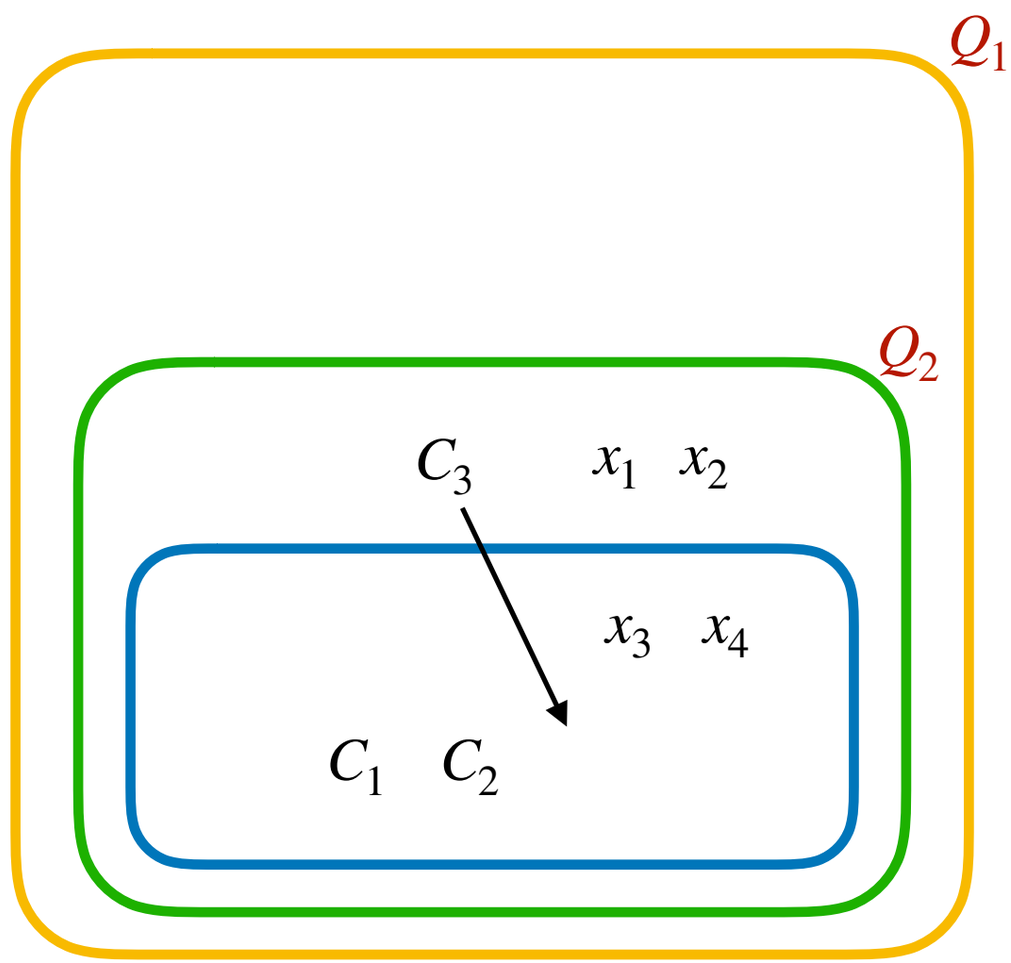

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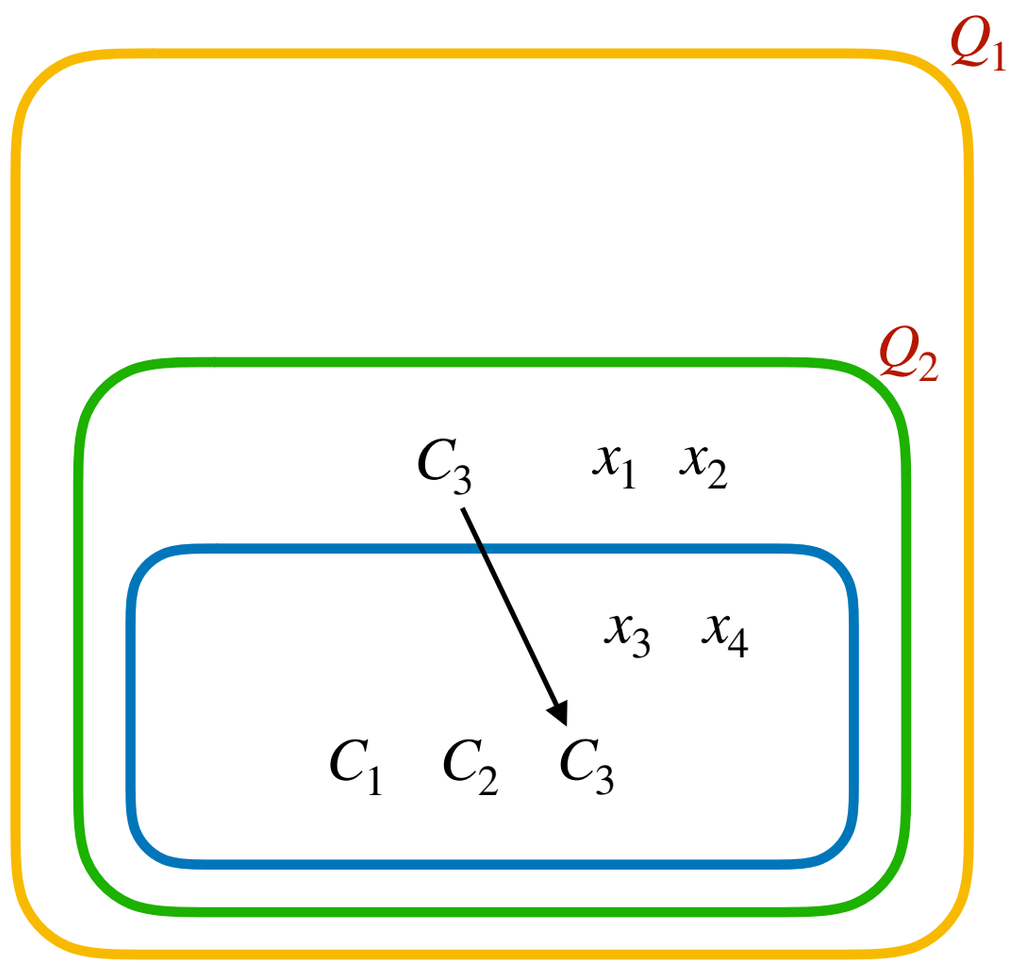
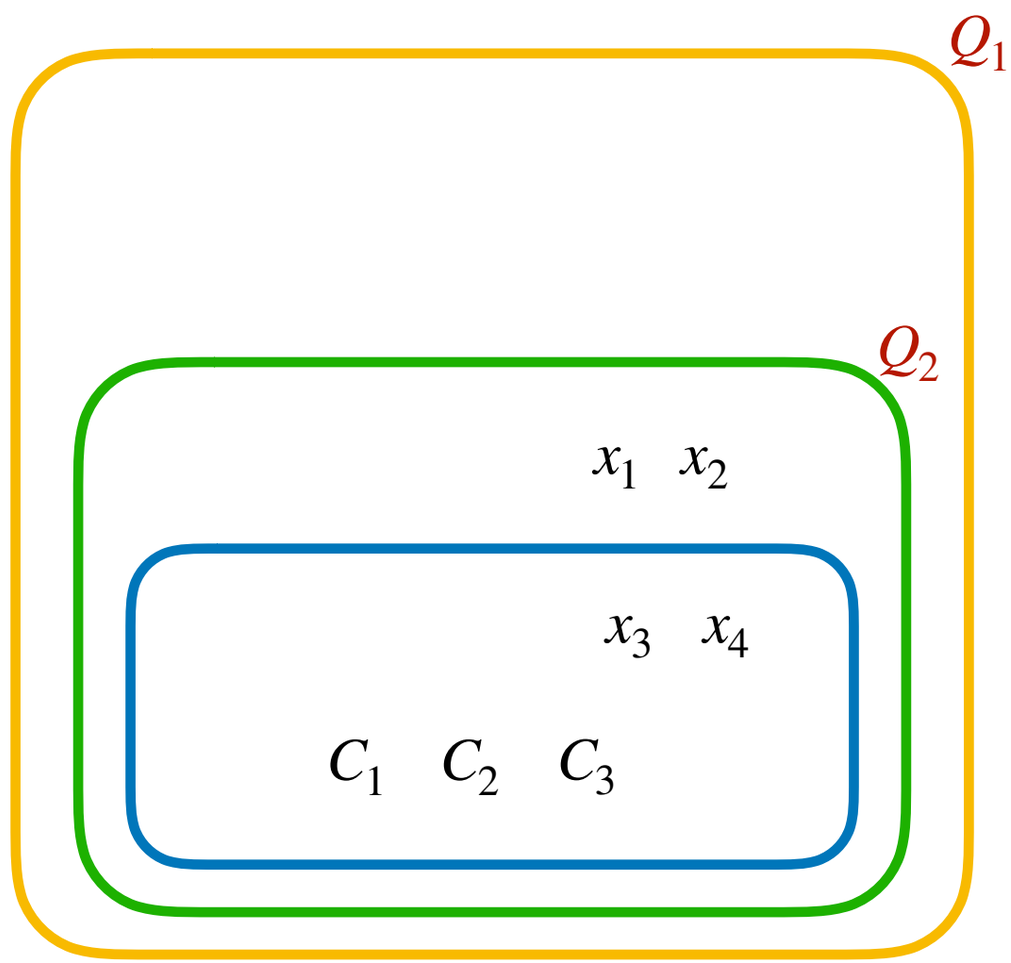
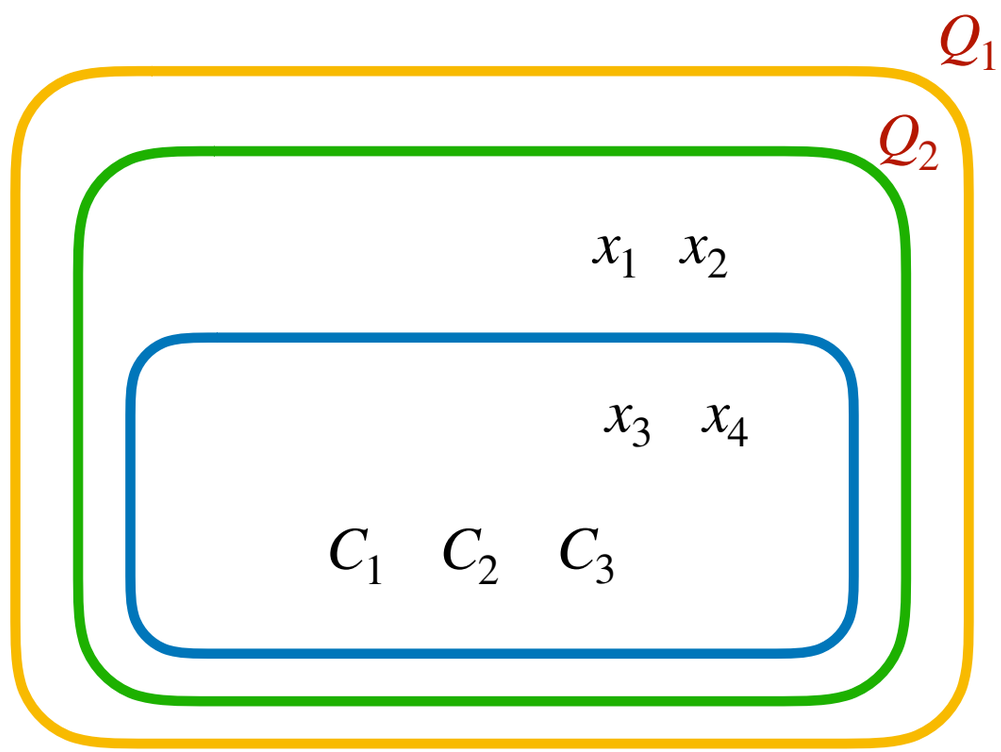


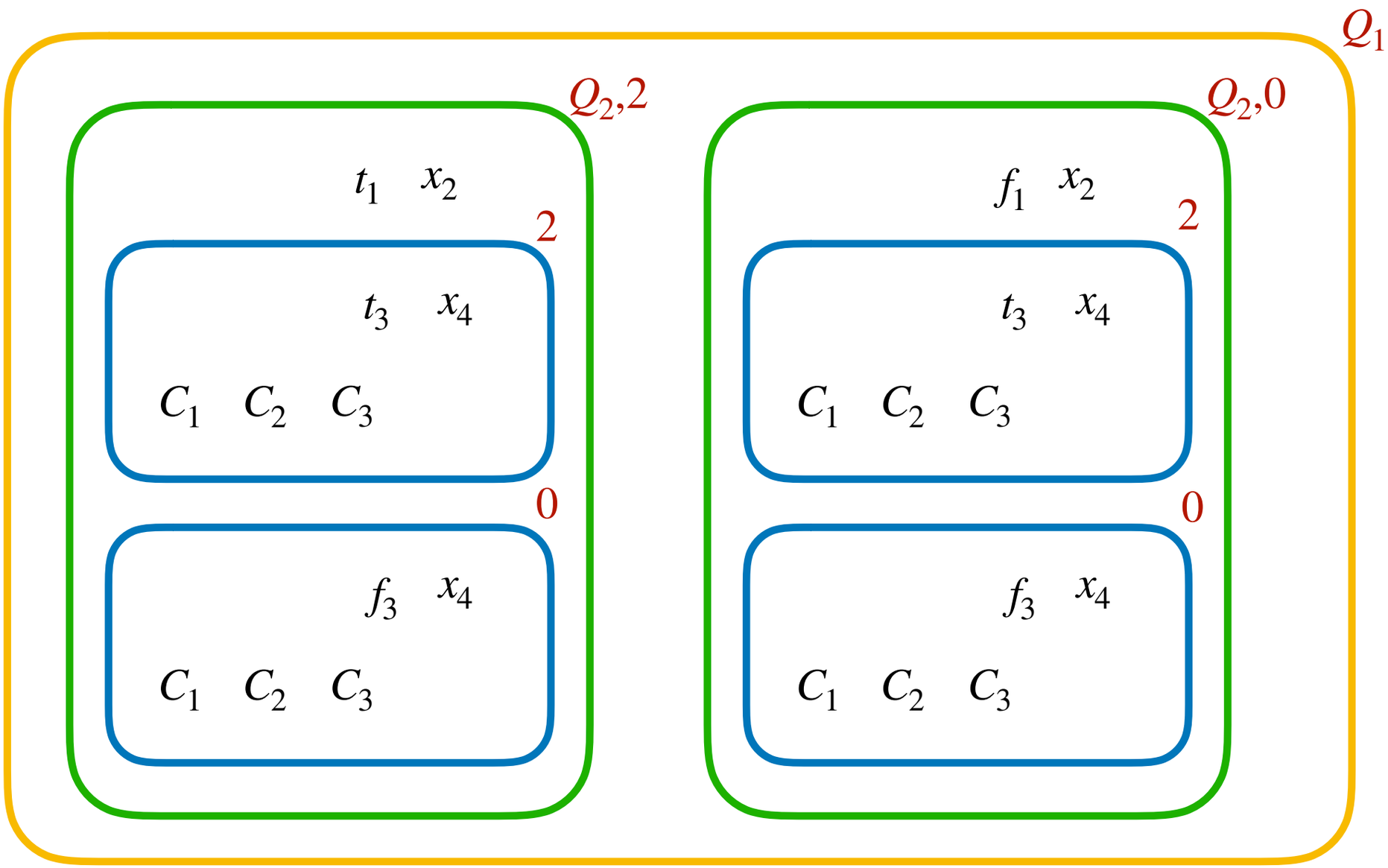
Sending Clauses Down



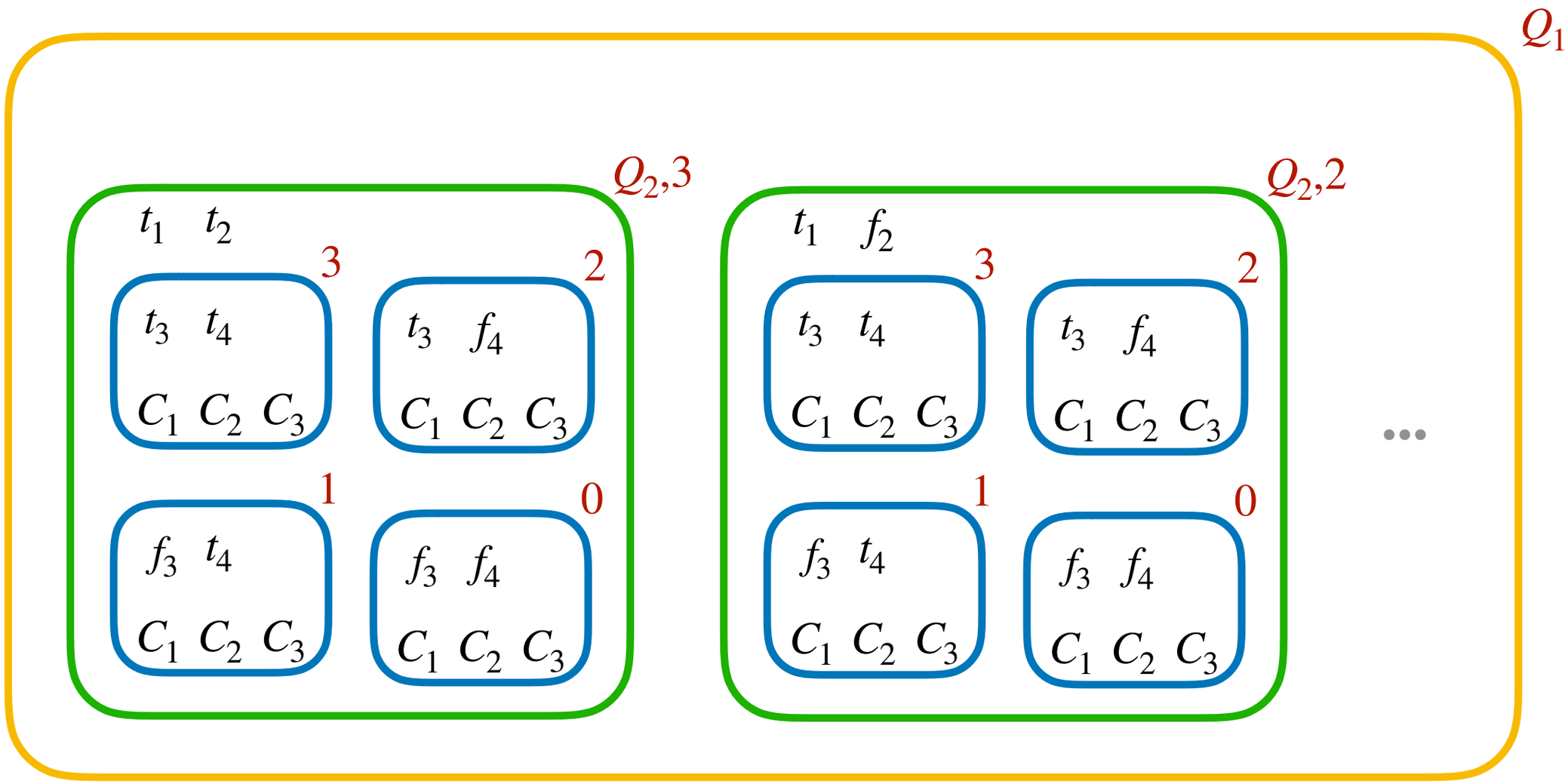
Assignments Generation



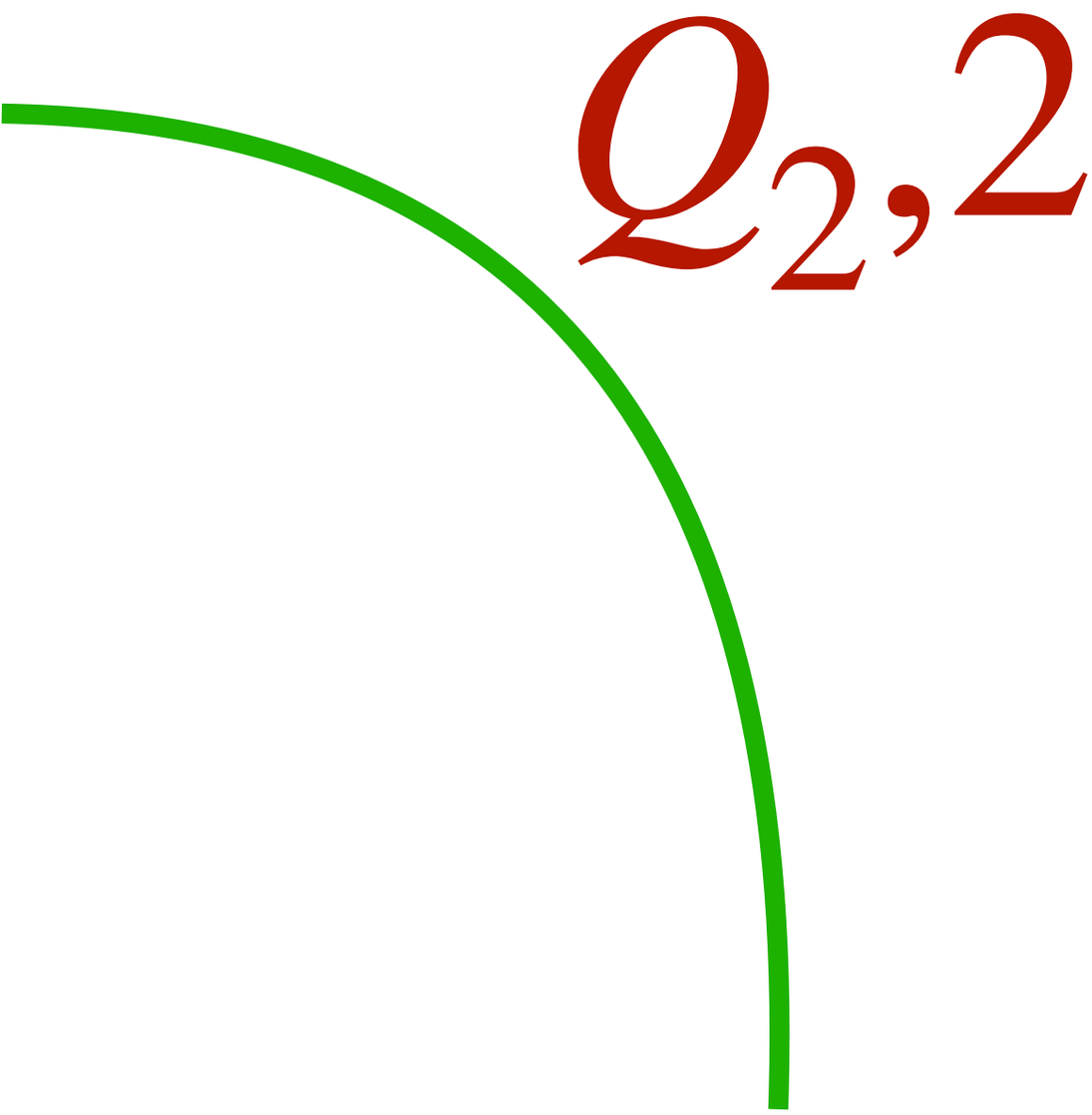
Assignments Generation



Assignments Generation



The Charge



The Charge

$$\mathcal{Q}_{2,2}$$

Block of quantifiers to be evaluated

The Charge

Position of this membrane in the "virtual tree"

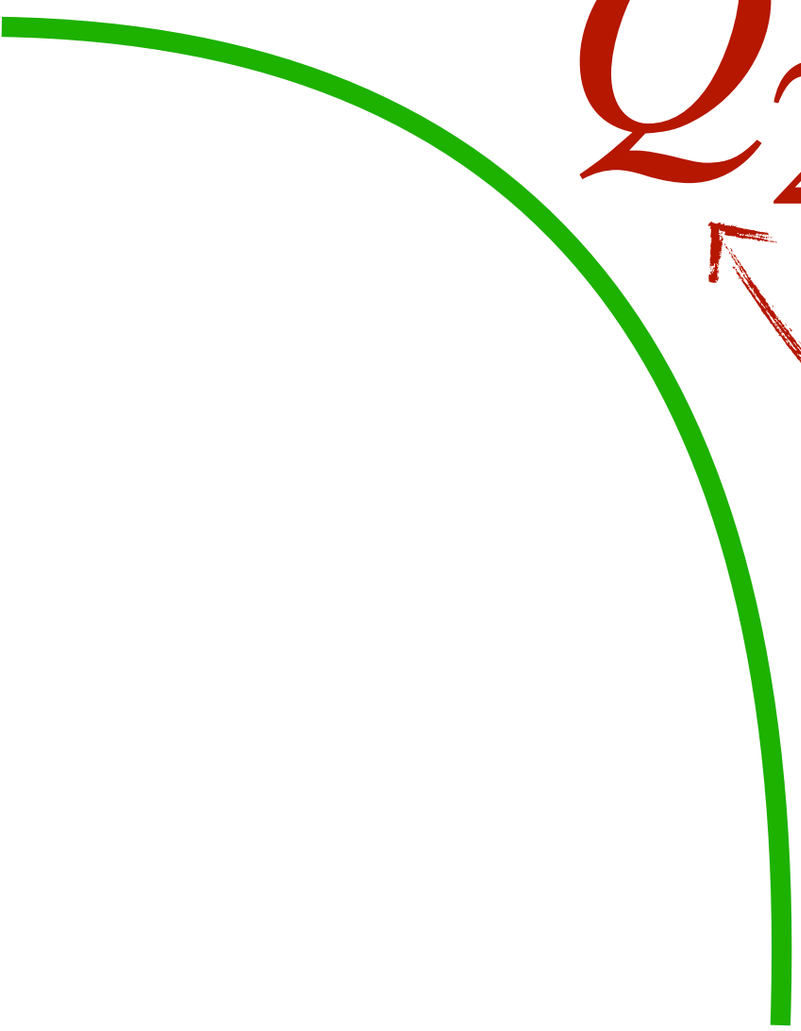
$$\mathcal{Q}_{2,2}$$

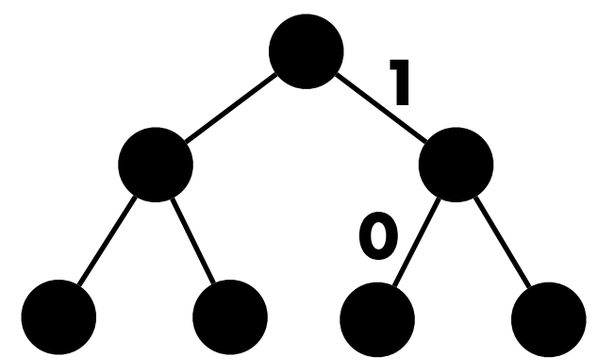
Block of quantifiers to be evaluated

The Charge

Position of this membrane in the "virtual tree"

$Q_{2,2}$



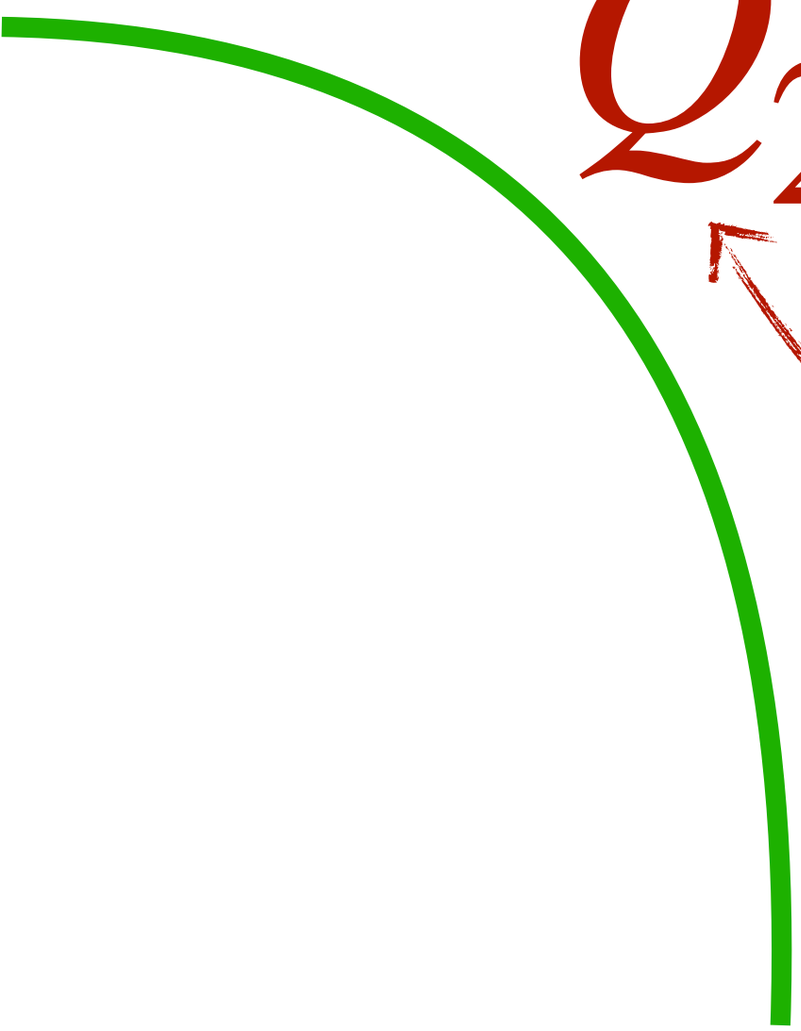


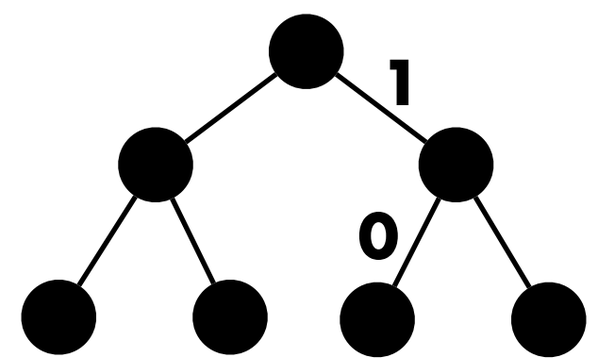
Block of quantifiers to be evaluated

The Charge

Position of this membrane in the "virtual tree"

$Q_{2,2}$





This membrane

Block of quantifiers to be evaluated

Formula Evaluation

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- As usual, use charges to “read” the assignment

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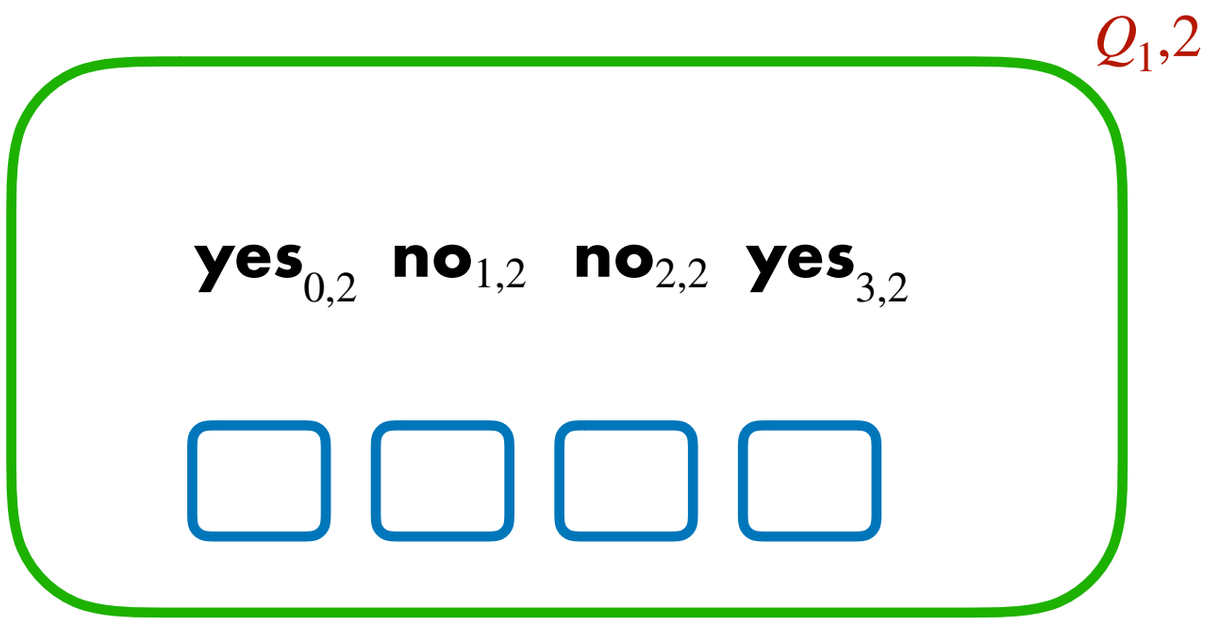
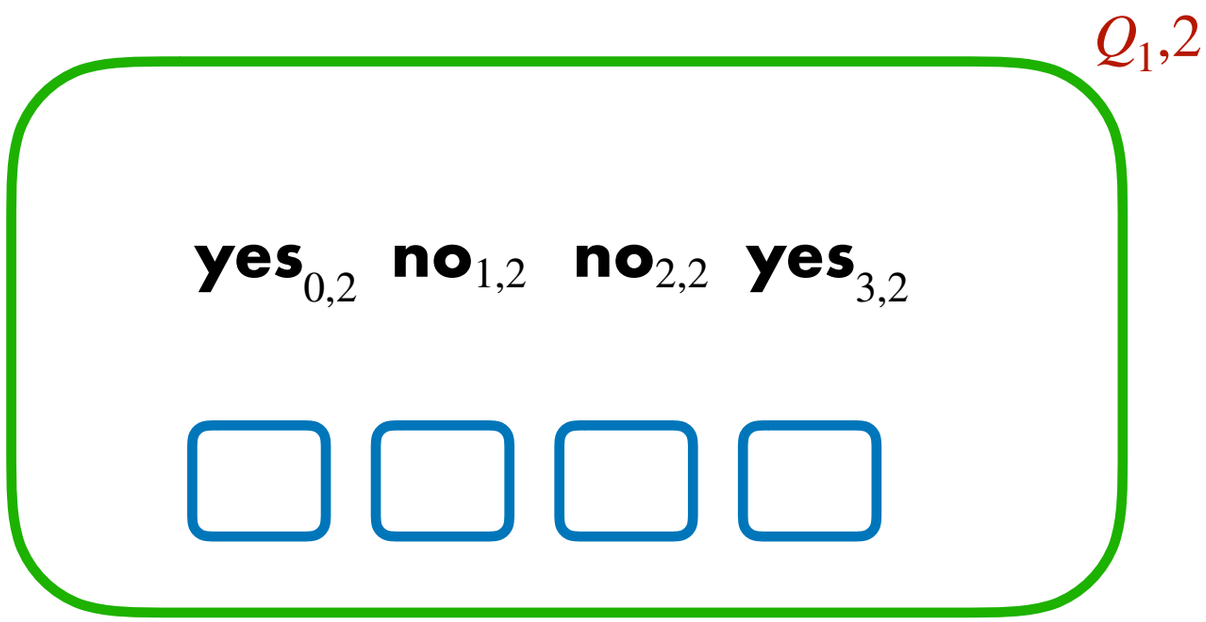
- We send out **yes** or **no** *+ subscript of the position in the "virtual tree"*

  + subscript of the depth in the "virtual tree"

Quantifiers evaluation



Quantifiers evaluation



Idea: evaluate sequentially inside the "virtual tree"

Quantifier Evaluation

$$Q_2 = \forall x_3 \exists x_4$$

$\textbf{yes}_{0,2}$ $\textbf{no}_{1,2}$ $\textbf{no}_{2,2}$ $\textbf{yes}_{3,2}$

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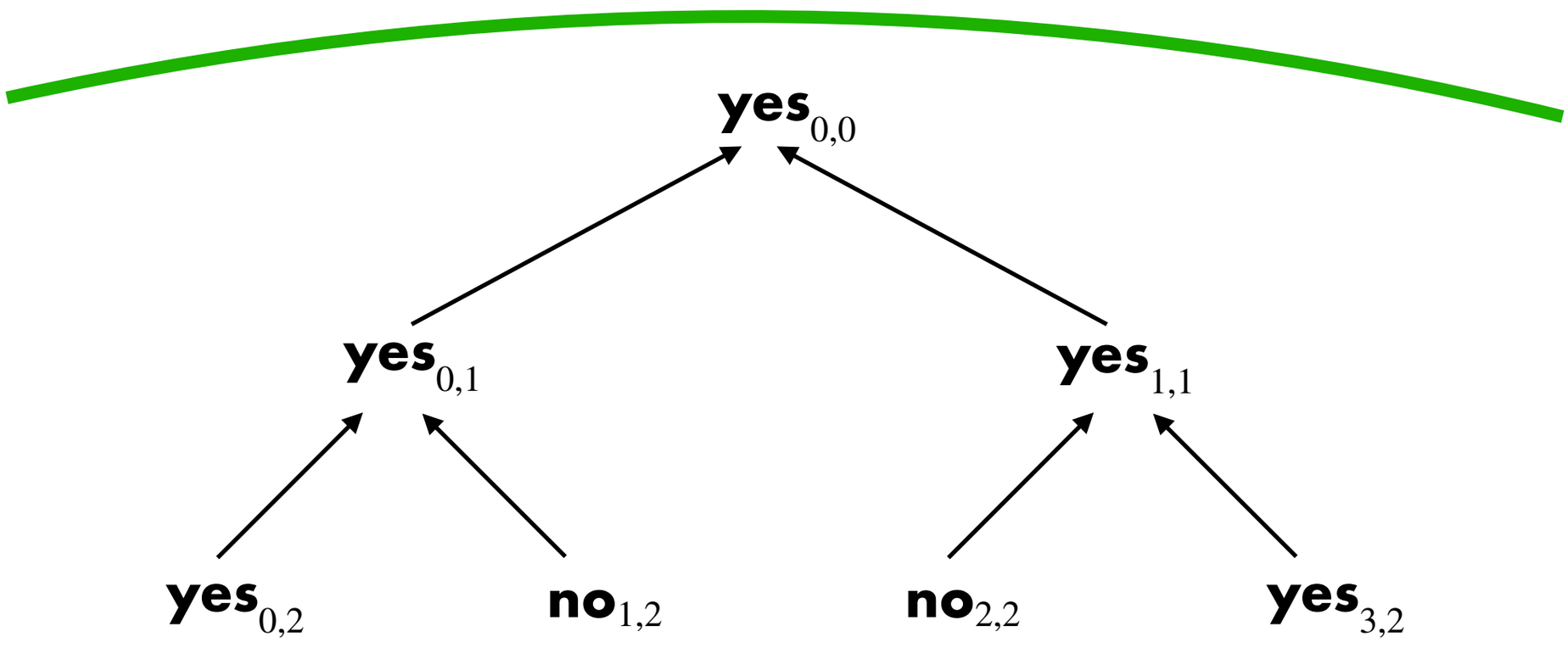

Quantifier Evaluation

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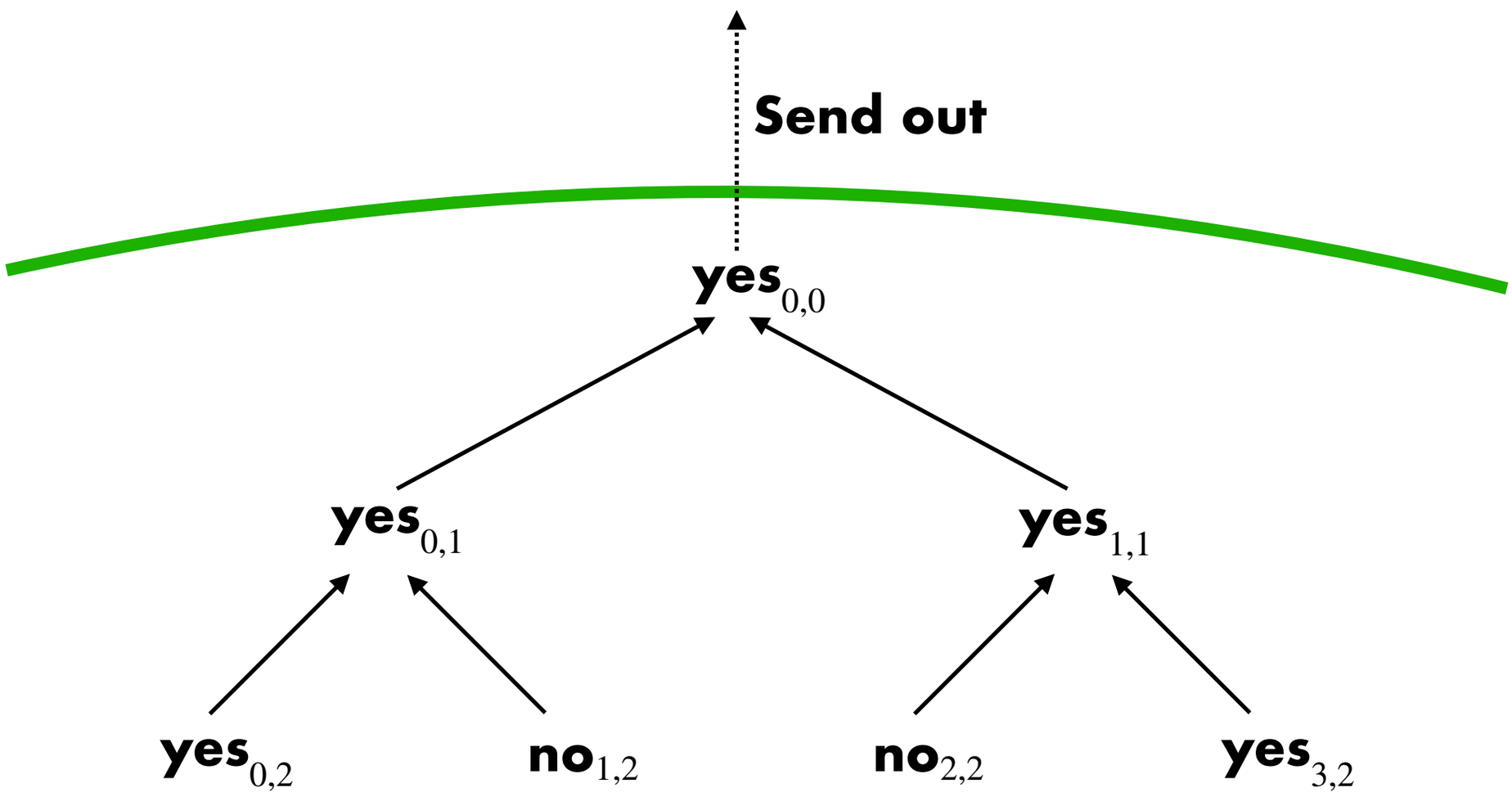

Quantifier Evaluation

Quantifier Evaluation

- Evaluation level by level in the “virtual tree”

Quantifier Evaluation

- Evaluation level by level in the “virtual tree”
- Each level evaluated from left to right

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Quantifier Evaluation

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- Each level evaluated from left to right
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+ lot of technical details omitted

Uniform families of P systems

Uniform families of P systems with active membranes

Uniform families of P systems with active membranes with charges with weak non-elementary division rules

Uniform families of P systems
with active membranes
with charges
with weak non-elementary division rules
working in polynomial time
can solve QSAT with depth

Uniform families of P systems
with active membranes
with charges
with weak non-elementary division rules
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can solve QSAT with depth

$$O\left(\frac{n}{\log n}\right)$$

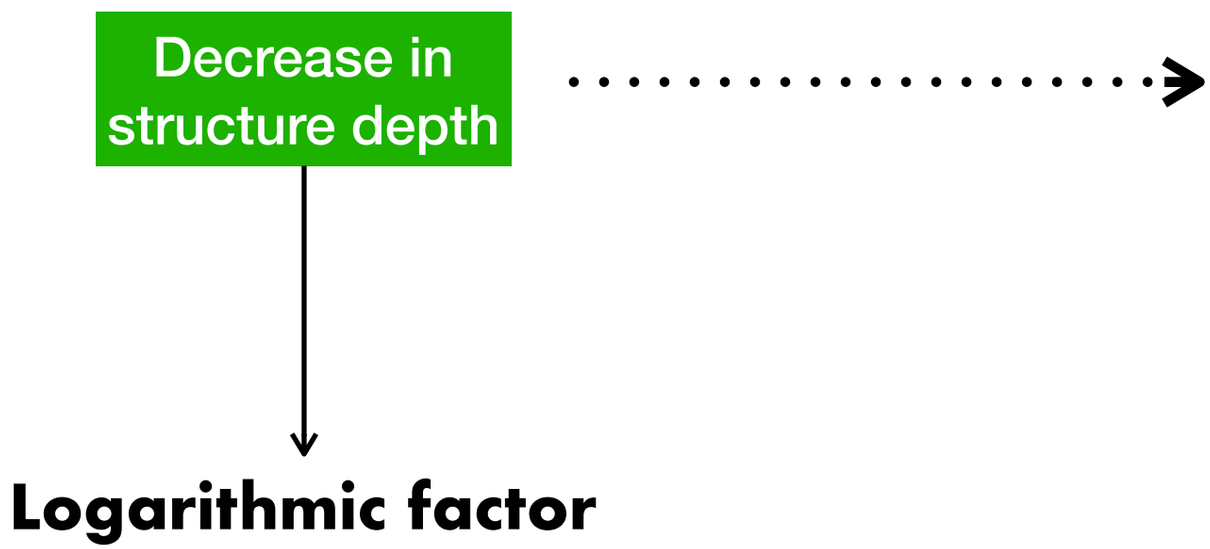
It's a Tradeoff!

Decrease in structure depth

↓

Logarithmic factor

It's a Tradeoff!



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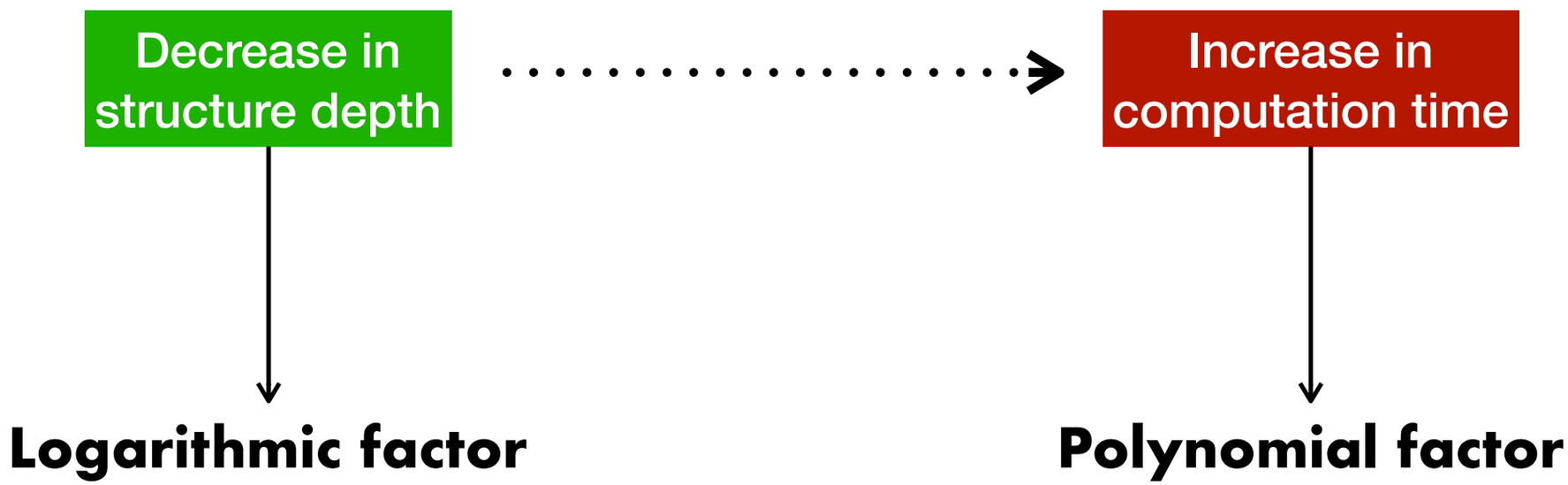

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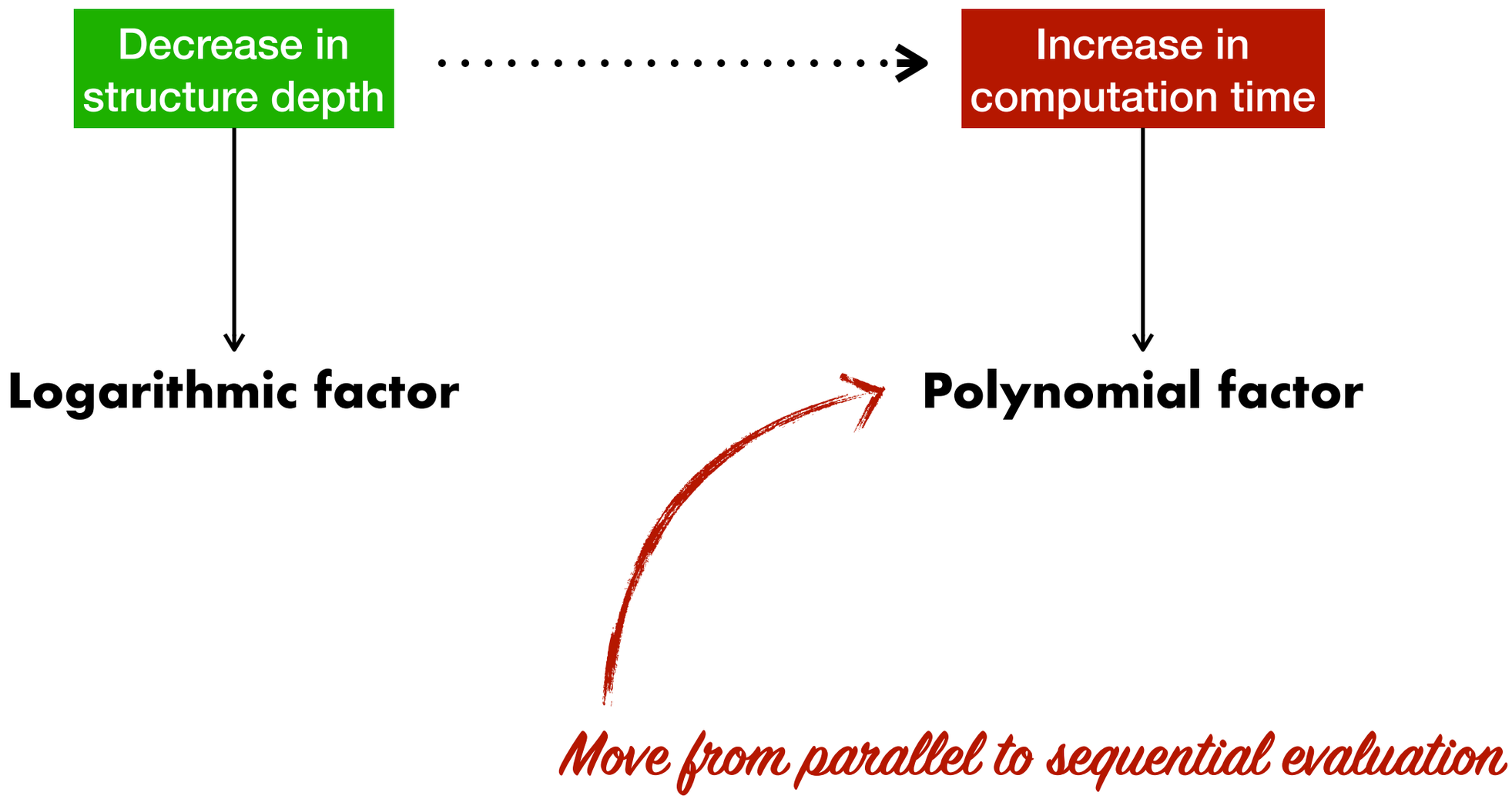

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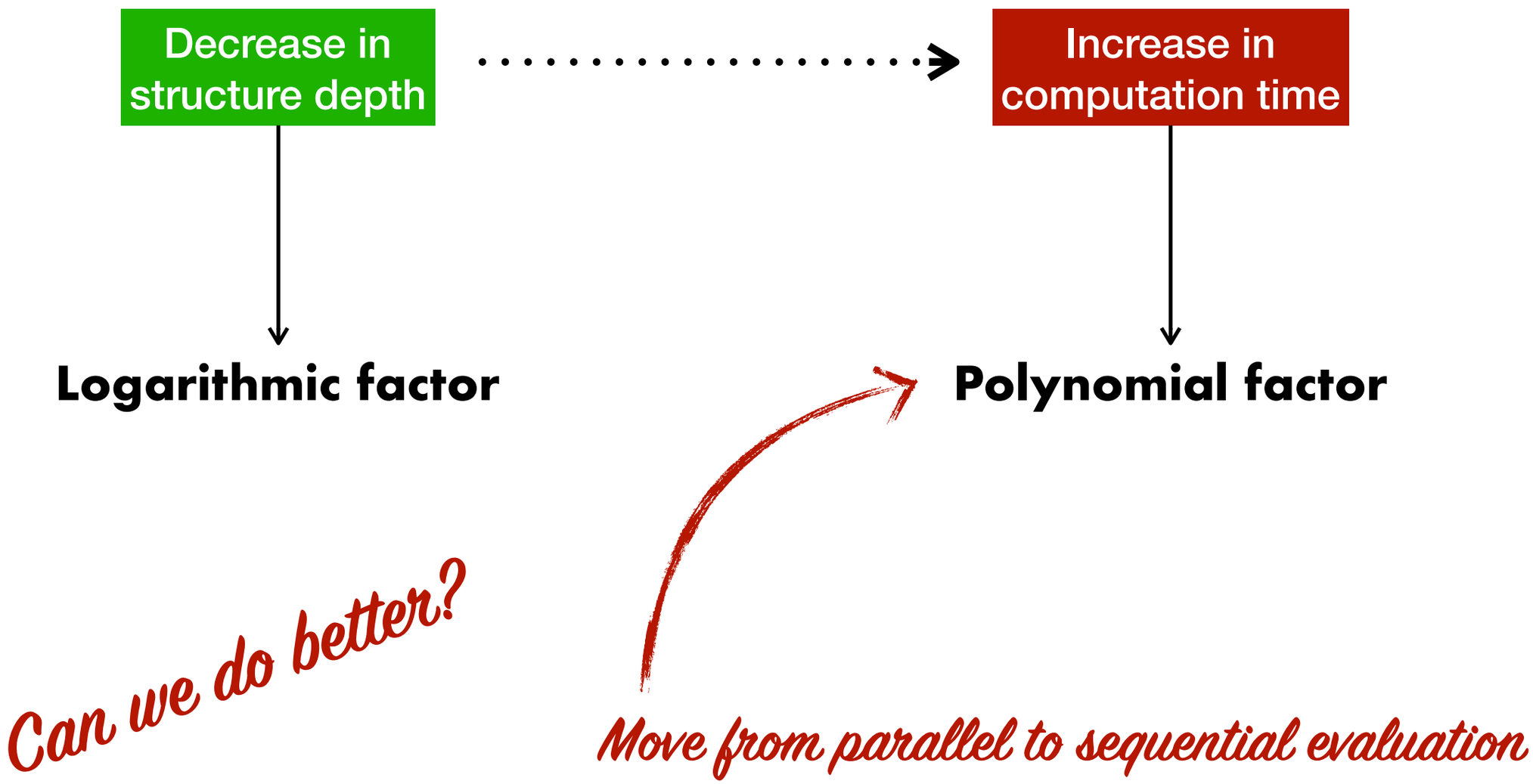

How **not** to Improve

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Can we do $O\left(\frac{n}{(\log n)^2}\right)$**?**

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NO

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Superpolynomial slowdown

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Can we do $O\left(\frac{n}{(\log n)^2}\right)$ **?**

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Superpolynomial number of charges

Superpolynomial slowdown

How **not** to Improve

Can we do $O\left(\frac{n}{(\log n)^2}\right)$ **?**

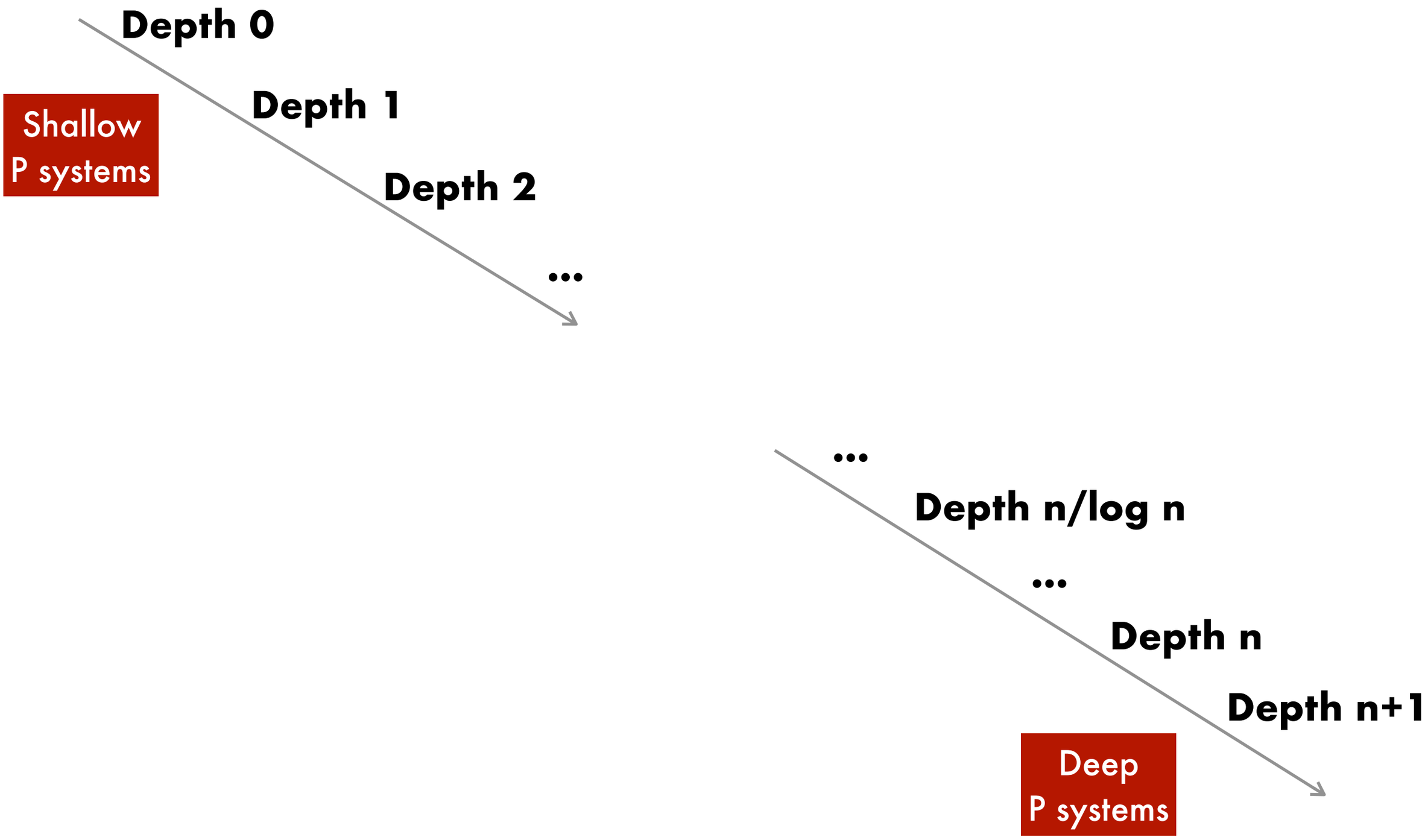
NO

Superpolynomial number of charges

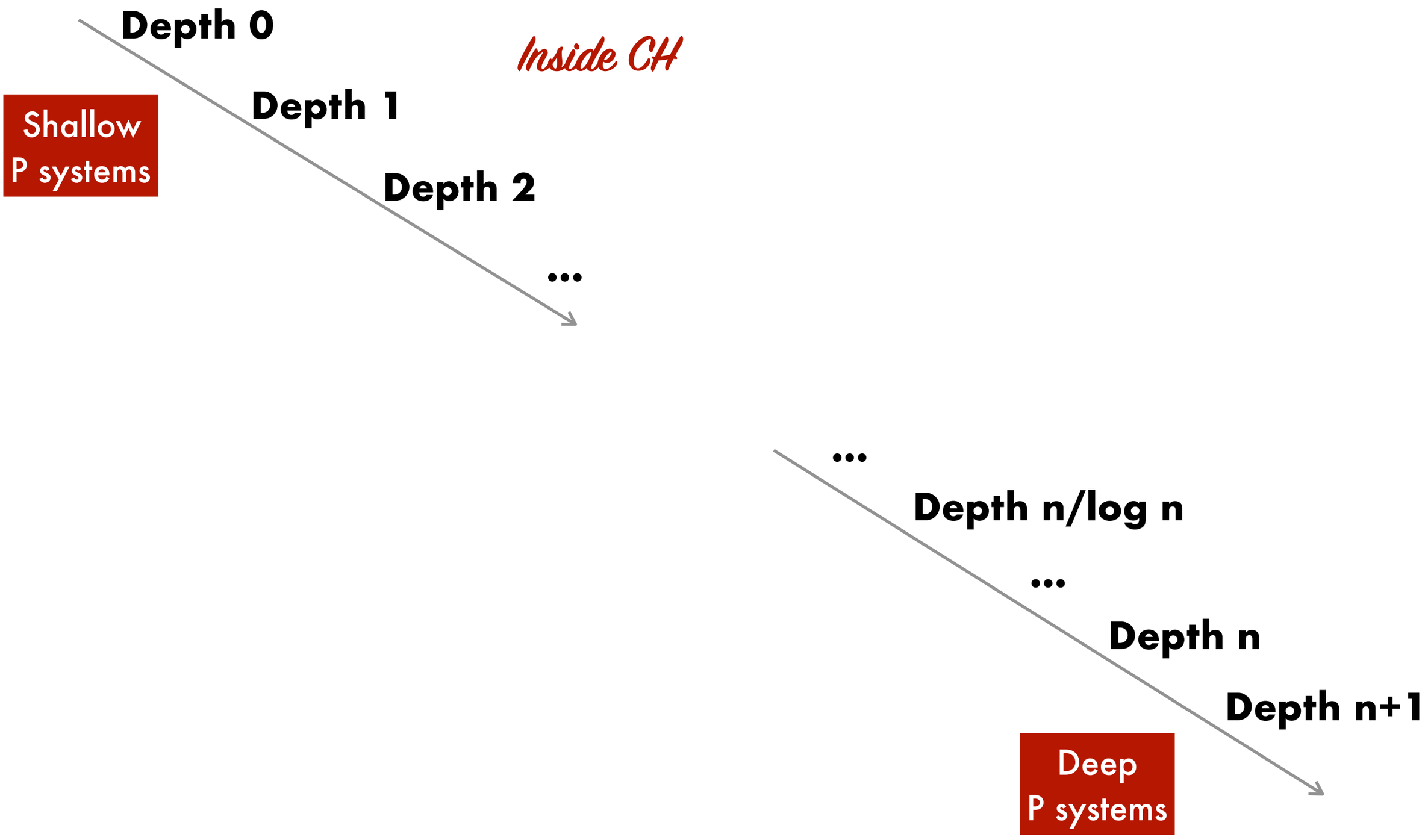
Superpolynomial slowdown

New methods are needed to improve this result

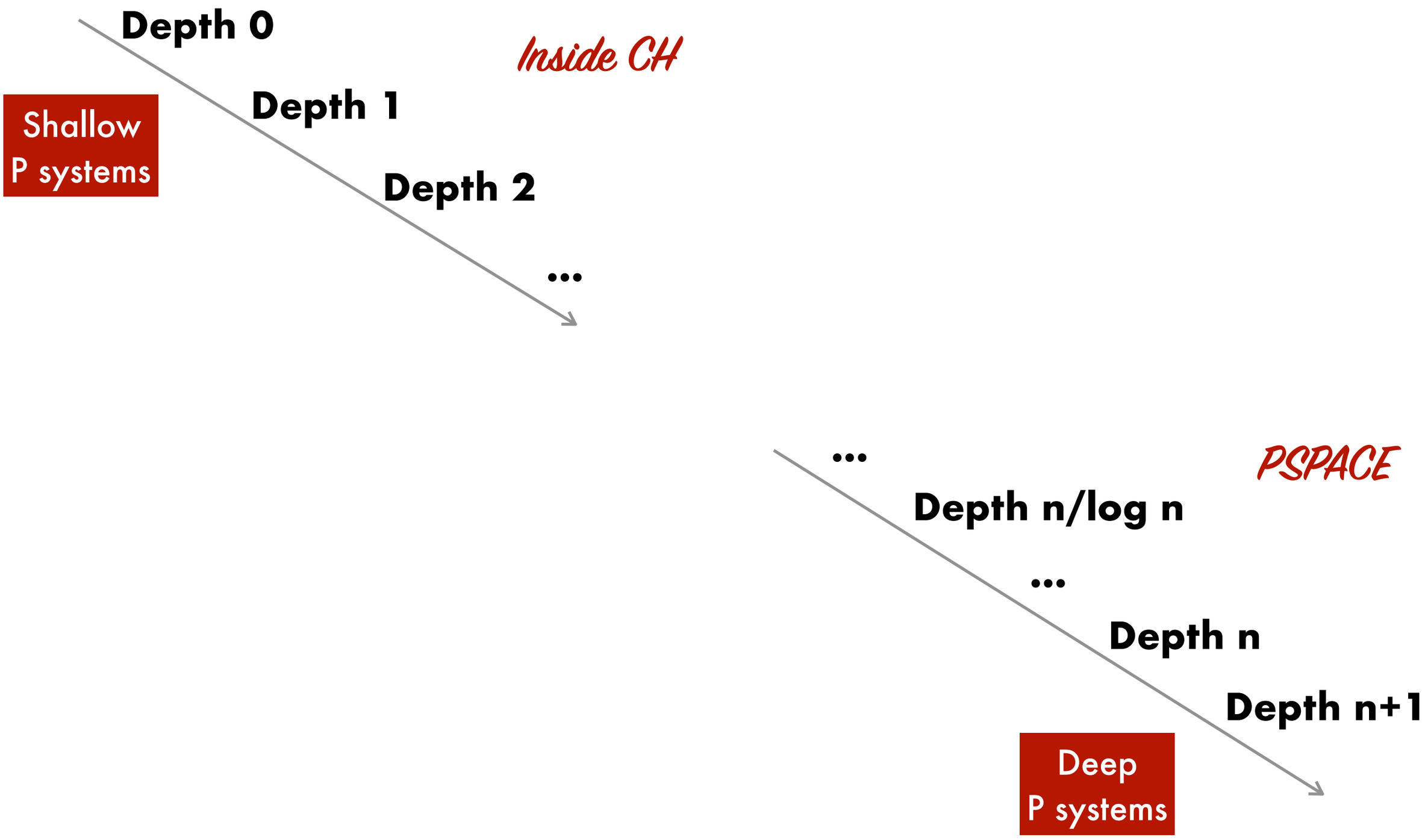
"The Gap", again



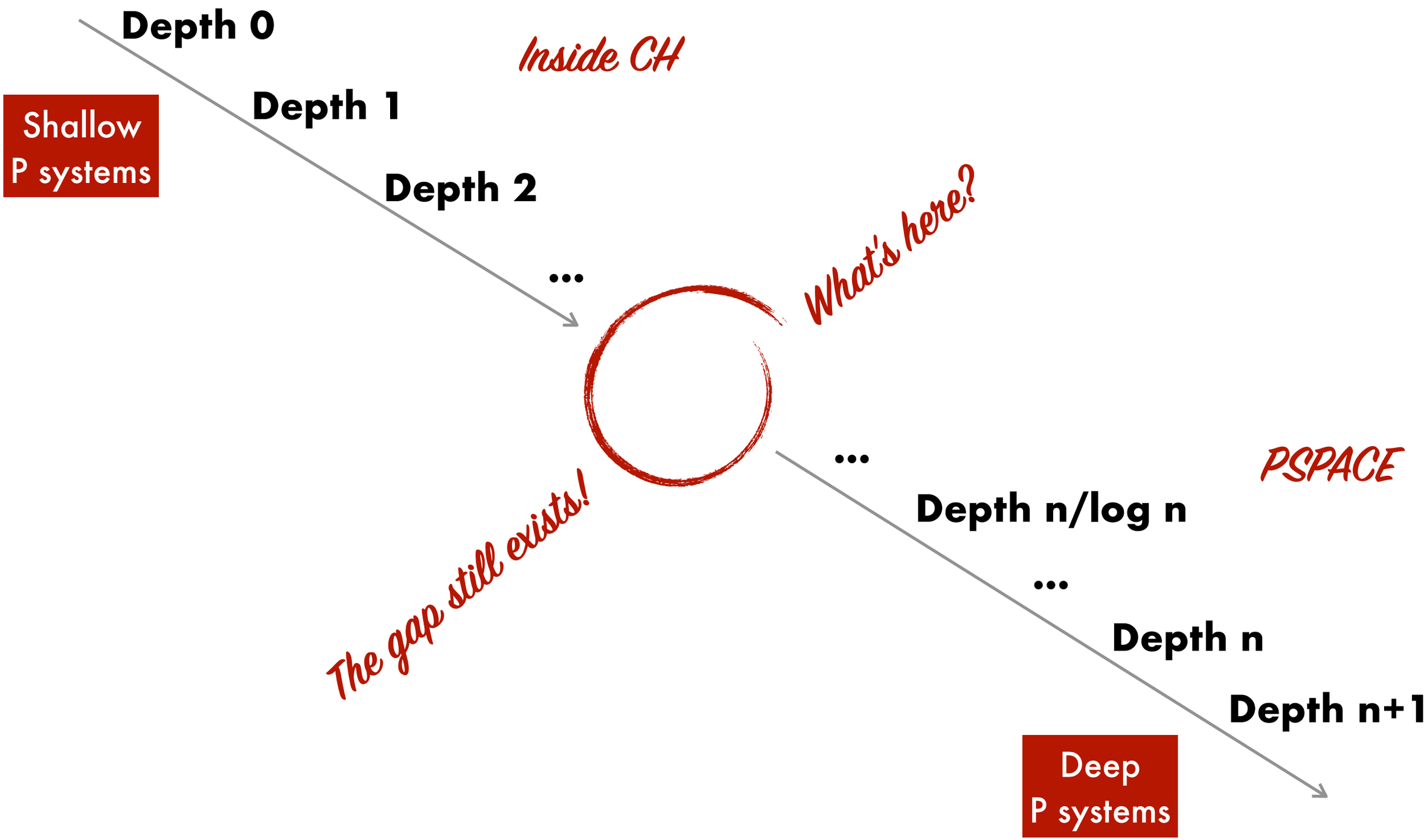
"The Gap", again



"The Gap", again



"The Gap", again



Gazing into the Gap

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- No “classical” class between CH and PSPACE

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- The power of logarithmic depth is unknown

Gazing into the Gap

- No “classical” class between CH and PSPACE
- The power of logarithmic depth is unknown
- What about polylogarithmic depth? Depth loglog?

Thank you for your attention