Modeling Plant Development with M Systems

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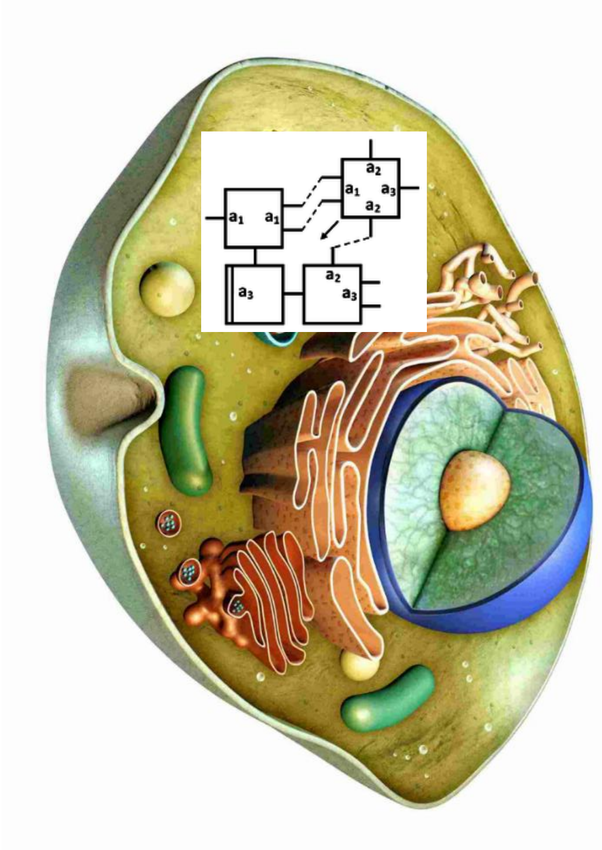
Motivation

- Create a computational model with focus at basic morphogenetic phenomena such as:
  - Growth
  - Homeostasis
  - Self-reproduction
  - Self-healing
- Simulate morphogenesis from scratch
  - Not to use atomic assembly units (cells)
  - Start from 1D/2D/3D primitives
  - Use self-assembly feature to create 3D cell-like forms



Morphogenetic systems (M systems)

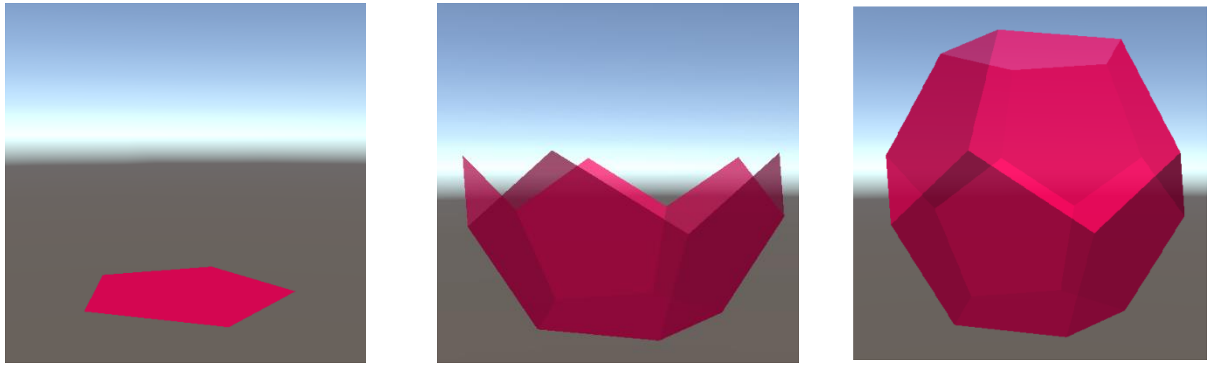
- Are based on principles of common membrane computing models
- Live in a 3D space (generally *d*D)
- Introduce explicit geometric features and self-assembly capabilities
  - Each elementary object has a fixed shape and position in space at any point in time
- Exhibit emergent behavior from local interactions
- Informed by tile assembly models
  - Polytopes and connectors like tiles and glues
- Use 3 types of objects:
  - Floating objects
  - Tiles
  - Protions (abtraction of biological "proteins")

Basic M system objects

- Floating objects
  - Small shapeless atomic objects floating freely within the environment
  - With a nonzero volume and specific position
- Tiles
  - Have their predefined size and shape (convex bounded polytopes)
  - Can stick together along their edges or at selected points called *connectors*
  - Can self-assembly into interconnected structures
- Protions
  - Are placed on tiles
  - Catalyze reactions of floating objects
  - Serve as „protion channels“ through (d–1)D tiles

Polytopic tile system in $\mathbb{R}^d$

- T = (Q, G, γ, d_g, S)
  - Q is the set of tiles – bounded convex *m*D polytopes
    (*m* ≤ *d*) with glues on their edges or selected points (connectors)
  - G is the set of glues
  - γ is the glue relation
  - d_g is the gluing distance
  - S is a finite multiset of ***seed*** tiles from Q distributed in space as an initial configuration



Formal definition

- Morphogenetic system ***M*** = (F, P, T, μ, R, σ)
  - F = (O, m, ρ, ε), the catalog of floating objects
    - O – the set of floating objects
    - m – mean mobility of each floating object
    - ρ – radius of interaction of each floating object
    - ε – concentration of each object in the environment
  - P – is the set of protions
  - T – is a polytopic tile system
  - μ – maps proteins to positions on M-tiles
  - R – is a set of reaction rules
  - σ – maps glue pairs to a multiset of floating objects produced when the binding is established

Reaction rules

- Are used for reactions and modifications of the M system during growth
- Four types of reaction rules:
  - Metabolic rules
  - Creation rules
  - Destruction rules
  - Division rules

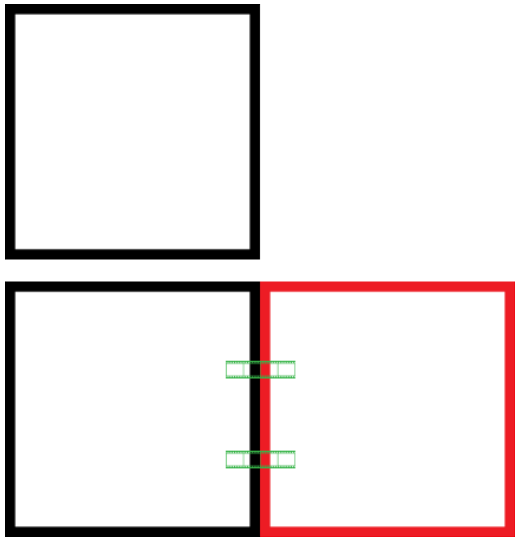
Metabolic rules

- A multiset of floating objects reacts and changes, or it is transported through a (*d*–1)D tile
- (*d*–1)-dimensional tiles have their sides marked "in" and "out", by convention

TYPE	RULE	EFFECT
SIMPLE	u → v	objects in multiset *u* react to produce *v*
CATALYTIC	pu → pv u[p → v[p [pu → [pv	objects in *u* react in presence of *p* to produce *v*; this variant requires both *u*, *v* at the side "out" of the tile; this variant requires both *u*, *v* at the side "in" of the tile;
SYMPORT	u[p → [pu [pu → u[p	passing objects in *u* through protion channel *p* to the other side of the tile
ANTIPORT	u[pv → v[pu	interchange of *u* and *v* through protion channel *p*

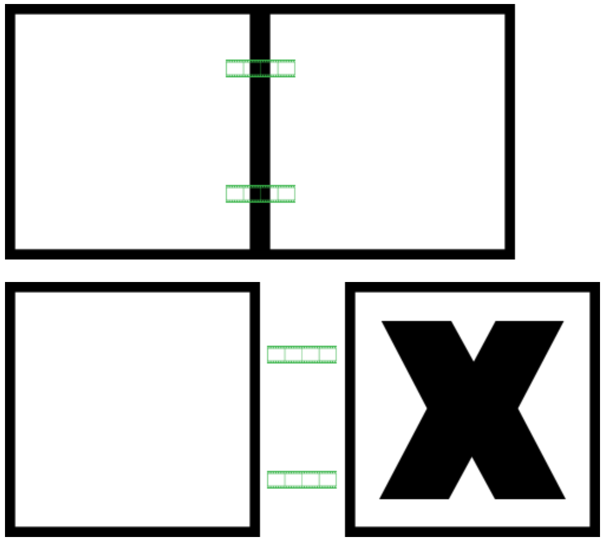
Creation rules

- Creates tile t while consuming the floating object in u
- Rule format: $u \longrightarrow v$



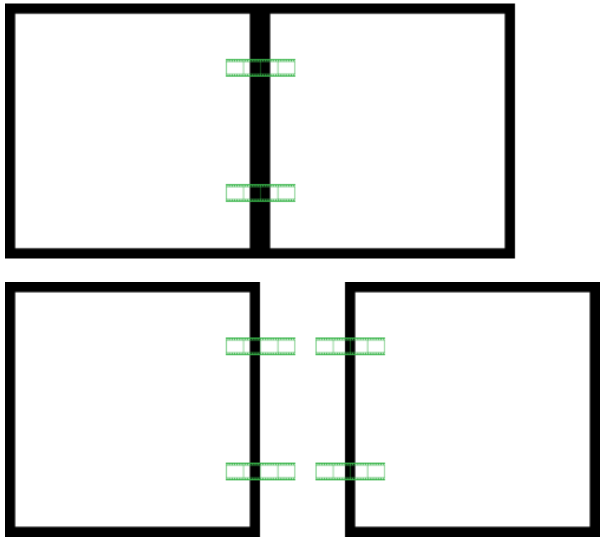
Destruction rules

- Tile *t* is destroyed in the presence of multiset of floating objects *u* which is consumed
- All connections from t to other tiles are released
- Rule format: $ut \longrightarrow v$



Division rules

- Two connectors with glues g, h get disconnected and the multiset x of floating objects is consumed
- Rule format: $g \overset{x}{\text{——}} h \rightarrow g, h$

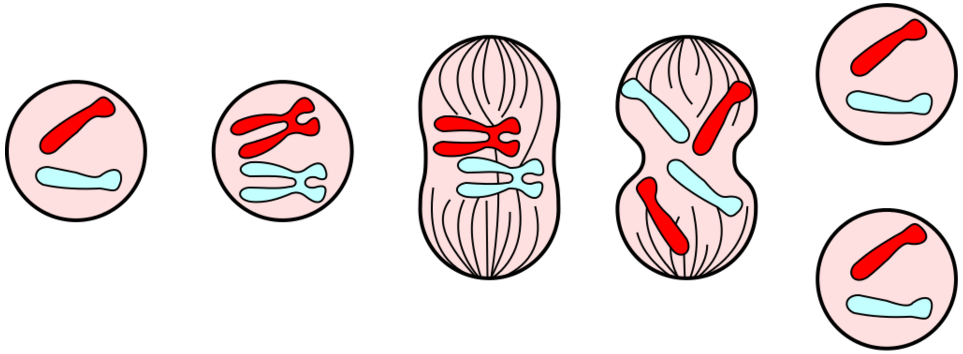


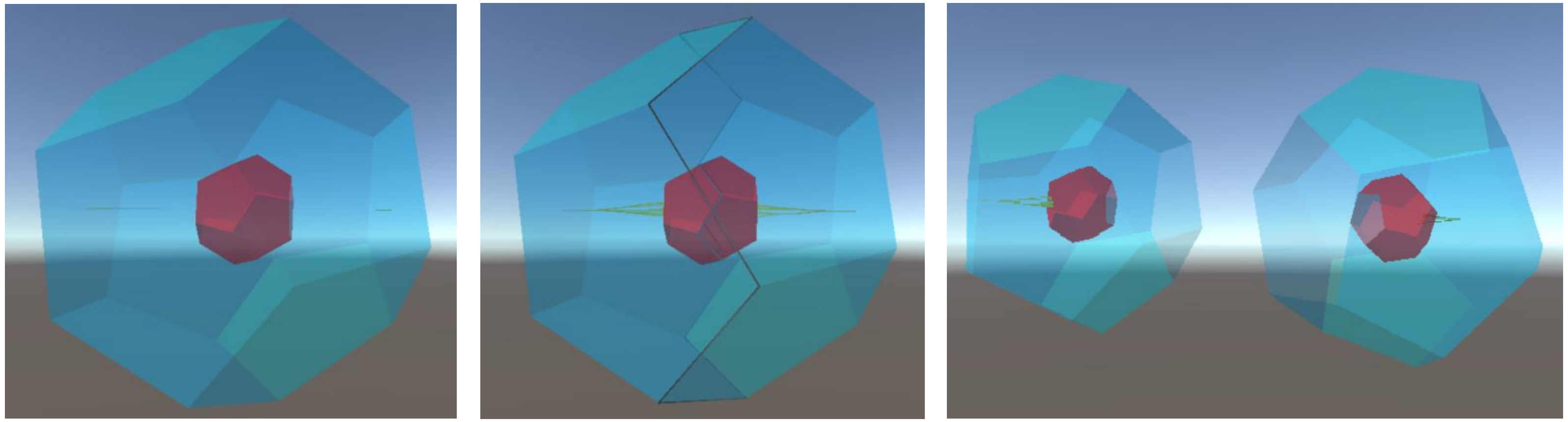
M system computation

- Initial configuration contains only *seed tiles* in *S* and random distribution of floating objects with concentration ε
- Computation takes place in discrete steps
- During each step, rules from *R* are applied in maximally parallel manner
  - Applicable rules are chosen randomly until no further rule is applicable
- Rules are applied in parallel to the actual configuration
- Each floating object changes its position randomly within its mobility perimeter
- A sequence of transitions between configurations is called a computation (nondeterministic)
- A computation ends when there is no longer any applicable rule

Example

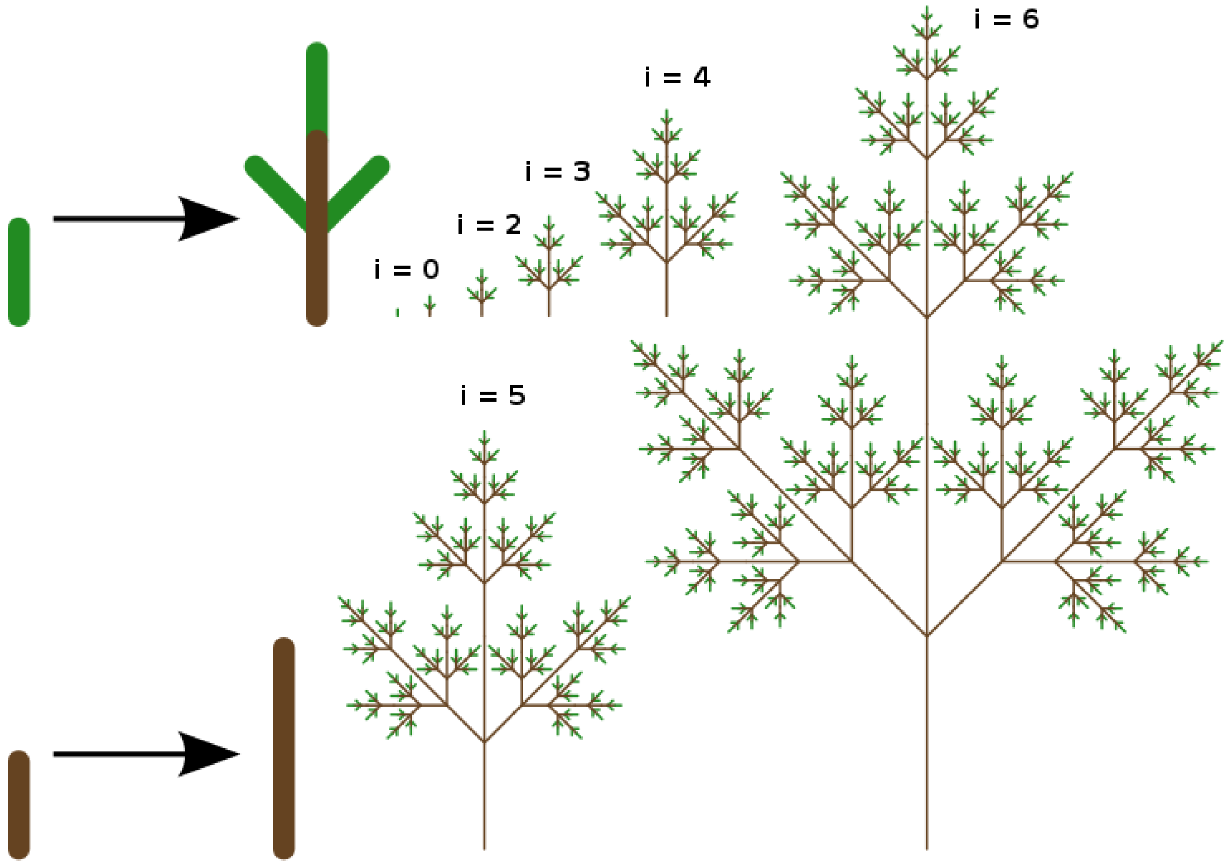
- Dynamics of cell replication controled by the cytoskoleton growth, simulated in an M system by our simulator `Cytos`





Lindenmayer systems (L systems)

- Parallel rewriting system
- Defined as a tuple $G = (\Sigma, \omega, P)$, where
  - Σ is the *alphabet*
  - ω is the *axiom* (initial state of the system)
  - P is a set of *production rules*
- Often interpreted by *"turtle graphics"*



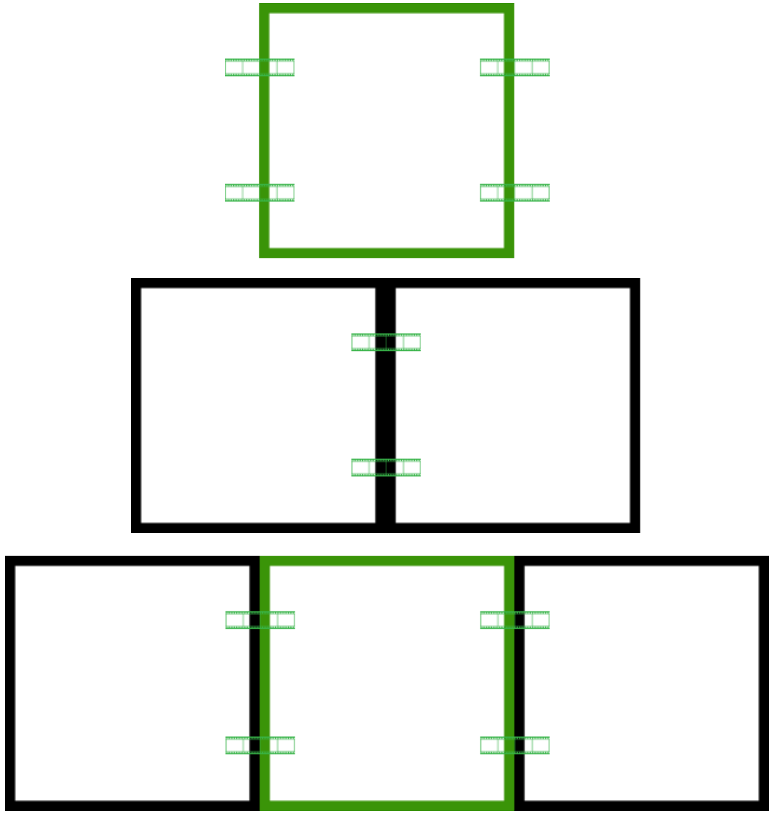
A comparation of L system and M system growth mechanism

- L system is more abstract and thus many growth processes generated by L system would appear impossible to do in M system
- Differences:

L system	M system
Any number of symbols (shape elements) can be generated without any limitations	Generation of shape elements controled by the presence of floating objects
Each string is graphically interpreted independently of the other, discontinuity is allowed	Each new shape is derived from the previous one only by adding/inserting/connecting new elements or disconnecting/deleting/pushing existing ones
Graphical interpretation is separated from the generation rules – it can be interpreted in different ways	Rules used during growth process have a specific geometric interpretation

Simulation of L systems by M systems

- L system can be stepwise simulated by an M system only under certain restrictions and require introducing *new rules*
- Insertion rule
  - Creates tile t while consuming floating objects in u.
  - Applicable only if there are two tiles with connectors such that t contains two compatible connectors at its opposite ens
  - Rule format: $u \longrightarrow t$



Simulation of L systems by M systems

Proposition 1.

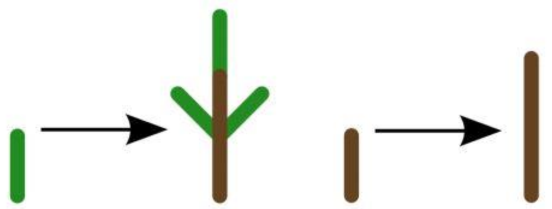
- Consider an L system $G = (\Sigma, \omega, P)$ with turtle graphics, with a set of variables $V \subseteq \Sigma$, where all rules in P are of the form

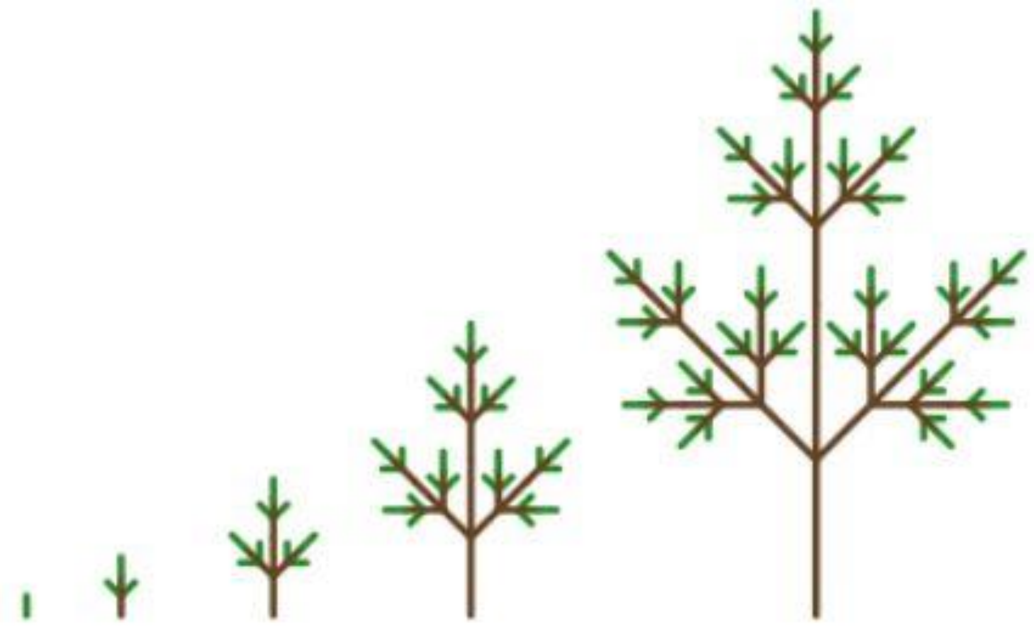
$$A \longrightarrow \{[\{+,-\}\, V^*] \cup V\}^* \, \{A\}\{[\{+,-\}\, V^*] \cup V\}^*, \quad \text{for } A \in V$$

- Then the growth of G can be stepwise simulated by an M system
- Each step of the L system can be simulated by a fixed number of steps of M system

Example of a tree growth by M system

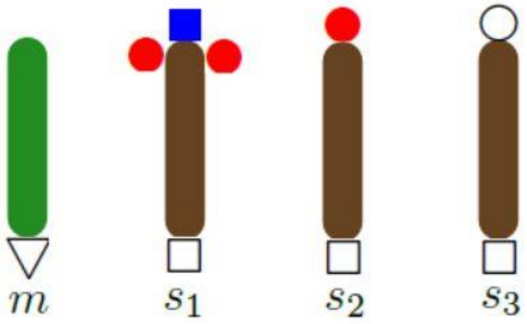
- Consider the L system $G_t = (\Sigma, M, P)$, where:
  - $\Sigma = \{M, S, +, -, [,]\}$
  - $P = \{M \to S[+M][-M]SM,\ S \to SS\}$,
  - M is green segment (leaves), S is brown segment (branches)
- Angle is fixed to 45°





Example of a tree growth by M system

- We construct an M system M_t such that each step of G_t is simulated by two steps of M_t
- Let T = (Q, G, γ, d_g, S) be a tile system, where:
  - G = { ●, ▽, ■, ○, □, x }
  - Q = {*m*, *s*1, *s*2, *s*3} are rods with surface glue x:
    - γ = { (▽, ■), (▽, ●), (□, ○), (□, ●), (□, ■) }
  - d_g = 0.1
  - S = {*m*}

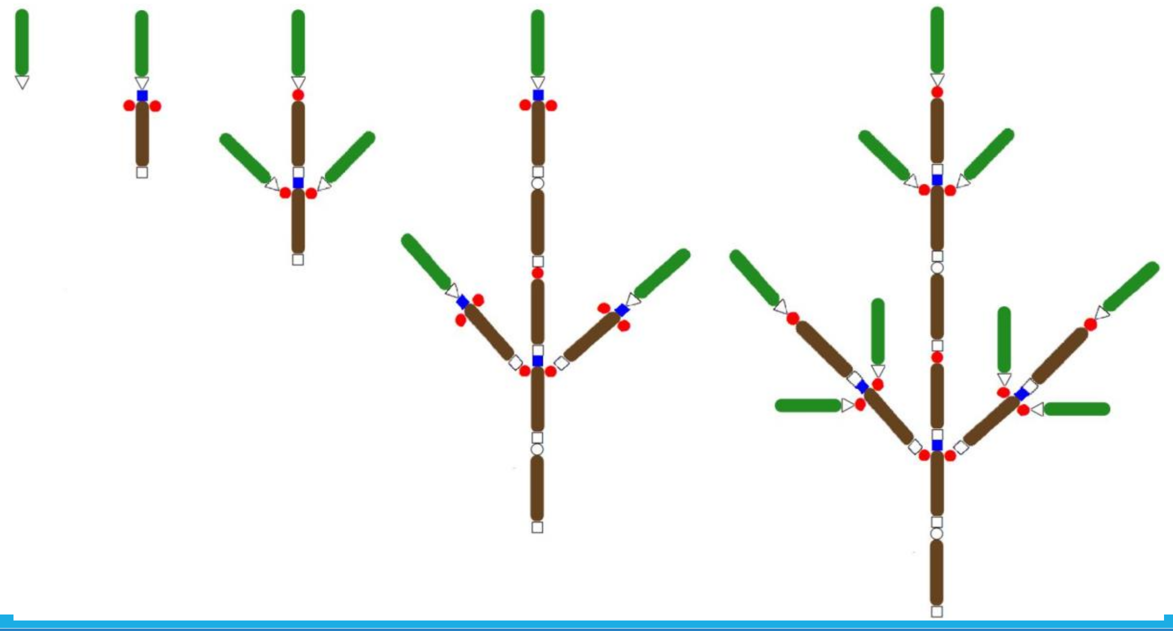


Example of a tree growth by M system

- M system M_t = (F, P, T, μ, R, σ)
  - F contains floating object *a, b* with high mobility
    - *a* is present in the environment with a high concentration
    - *b* with zero concentration
  - P is an empty set of protions
  - σ assigns to each glue pair $(g_1, g_2) \in \gamma$ the empty multiset
  - R contains the following rules:
    - Metabolic rules: $a \to b$, $b \to a$
    - Creation rules: $aaa \to s1$, $aaa \to s3$, $bbb \to m$
    - Insertion rules: $aaa \to s1$, $aaa \to s3$, $bbb \to s2$

Example of a tree growth by M system

- Using these rules, the M system M oscillates between two states
- One with floating objects *a* and the other with floating objects *b* in the environment
- Rods *s*1 and *s*3 can be created only at an odd step, while rods *s*2 and *m* at an even step
- One step of L takes two steps of M



Thank you, any questions?

For more information and free download of the M system simulator and the visualization engine please consult Morphogenetic systems download page:

http://sosik.zam.slu.cz/msystem/

or use QR code