A Semantic Investigation of Spiking Neural P Systems

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Aim and Contribution

- We present a denotational semantics $[\![\cdot]\!]$ for a language $\mathcal{L}_{SNP}$ inspired by the spiking neural P (SN P) systems [Ionescu, Păun and Yokomori - 2006]
  - At syntactic level $\mathcal{L}_{SNP}$ provides constructions for specifying: neurons and synapses, rules with time delays
  - The denotational semantics $[\![\cdot]\!]$ for $\mathcal{L}_{SNP}$ is designed with metric spaces and continuations
- We provide a Haskell implementation of $[\![\cdot]\!]$

  `http://ftp.utcluj.ro/pub/users/gc/eneia/cmc19`

Aim and Contribution

- SN P systems - a class of P systems inspired from the way neurons communicate by means of spikes [Păun - 2007]
  - Equivalent in computational power to Turing machines
  - Able to solve NP-complete problems in polynomial time

- We investigate the behavior of SN P systems using methods specific of programming languages semantics
  - Syntax of $\mathcal{L}_{SNP}$ is specified in BNF
    - $\mathcal{L}_{SNP}$ constructions are called statements or programs
  - Semantics of $\mathcal{L}_{SNP}$ is described in denotational style

Aim and Contribution

- Our denotational semantics $[\![\cdot]\!]$ describes accurately
  - The structure of SN P systems: neurons, synapses, spikes
  - The behavior of SN P systems:
    - Time delays between firings and spikings
    - Non-deterministic behavior and synchronized functioning
- $[\![\cdot]\!]$ is the first denotational (compositional) semantics for this combination of concepts, specific of SN P systems

Principles of Denotational Sematics (mathematical or Scott-Stratchey semantics)

- *Language constructions denote values from a mathematical domain of interpretation*

$$[\![\cdot]\!] : \mathcal{L} \to \mathbf{D}$$

- *Definitions are compositional*

$$[\![\cdots x_1 \cdots x_2 \cdots]\!] = \cdots [\![x_1]\!] \cdots [\![x_2]\!] \cdots$$

- Various options in designing **D** and $[\![\cdot]\!]$ for a given $\mathcal{L}$
  - Classic (order-theoretic) domains vs metric spaces
  - Direct semantics, continuations

Metric Spaces vs Order-Theoretic Domains

- The purpose of domain theory is to give models for spaces on which to define computable functions [Scott - 1982]

- In classic domains (order-theoretic domains)
  - One works with least fixed points of continuous functions
  - Not all elements are comparable, the order is partial

- Metric spaces employ additional information
  - One can (compare and even) measure the distance between any two elements of a metric space
  - Contracting functions on complete metric spaces have unique fixed points (Banach's theorem)

Metric Spaces vs Order-Theoretic Domains

- Domain theory was initiated by [Scott - 1976, Scott - 1982]
  - Scott's key construction - a solution of the 'equation'

$$\mathbf{D} \cong \mathbf{D} \to \mathbf{D}$$

- We offer a semantic description of SN P systems based on a domain of continuations

$$\mathbf{D} \cong \mathbf{K} \to \mathbf{K} \qquad \mathbf{K} \cong \cdots \mathbf{D} \cdots$$

- Following [De Bakker and De Vink - 1996] we employ the mathematical methodology of metric semantics
  - Traditional (direct) concurrency semantics may not work for the complex interactions specific of MC and SN P systems

Continuation Semantics for Concurrent Languages

- In Continuation-Passing Style (CPS) control is passed explicitly in the form of continuations [Appel - 2007]

- We need a domain of continuations which can store computations (between firings and spikings) in CSC style [Todoran - 2000, Ciobanu & Todoran - 2014]

$$\mathbf{D} \cong \mathbf{K} \to \mathbf{K} \qquad\qquad \mathbf{K} \cong \cdots \mathbf{D} \cdots$$

- In previous work we investigated MC concepts by using a simple domain of continuations

G. Ciobanu and E.N. Todoran, Denotational Semantics of Membrane Systems by using Complete Metric Spaces, Theor. Comput. Sci., 2017.

Syntax of $\mathcal{L}_{SNP}$

Definition (Syntax of $\mathcal{L}_{SNP}$)

(a) *(Statements)* $x(\in X) ::= a \mid \mathrm{send}(y,\xi) \mid x \parallel x$

$y(\in Y) ::= a \mid y \parallel y$ *(obviously, $Y \subseteq X$)*

(b) *(Rules)* $r(\in Rs) ::= r_\epsilon \mid \varrho, r$

$\varrho(\in R) ::= E/w \to x; t \mid w \to \lambda,$

(E is a regular expression over O, $w \neq []$, $t \geq 0, t \in \mathbb{N}$)

(c) *(Neuron declarations)*

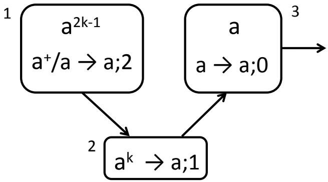
$d(\in ND) ::= \mathrm{neuron}\, N\,\{\, r \mid \xi \,\}$ $D(\in NDs) ::= d \mid d, D$

(d) *(Programs)* $\rho(\in \mathcal{L}_{SNP}) ::= D, x$ *(x executed by first neuron in D)*

- $(a \in)O$ - alphabet of spikes/objects (several types of spikes)
- $(N \in)Nname$ - set of neuron names
- $(w \in)W = [O]$ - set of multisets over O
- $(\xi \in)\Xi = \mathcal{P}_{fin}(Nname)$ - finite sets of neuron names
- Extended rules - a statement x is able to produce more than one spike
- $\mathrm{send}(y,\xi)$ is specific of $\mathcal{L}_{SNP}$
  (Instead of W and Ξ we could use O^* and $Nname^*$)

An $\mathcal{L}_{SNP}$ program and its intuitive behavior

$$\rho_1 = (D_1, x_1)$$
$$x_1 = \mathrm{send}(\langle a^{2k-1}\rangle, \{N_1\}) \parallel \mathrm{send}(a, \{N_3\})$$
$$\begin{aligned} D_1 = \; & \text{neuron } N_0 \{ r_\epsilon \mid \{N_1, N_2, N_3\} \}, \\ & \text{neuron } N_1 \{ a^+/[a] \to a; 2 \mid \{N_2\} \}, \\ & \text{neuron } N_2 \{ [a^k] \to a; 1 \mid \{N_3\} \}, \\ & \text{neuron } N_3 \{ [a] \to a; 0 \mid \{N_0\} \} \end{aligned}$$



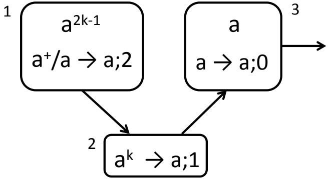
[Ionescu, Păun and Yokomori – 2006]
SN P system Π_1

$\{(N_0, [\,]), (N_1, [a, a, a]), (N_2, [\,]), (N_3, [a])\}$
$\Rightarrow \{(N_0, [a]), (N_1, [a, a, a]), (N_2, [\,]), (N_3, [\,])\}$
$\Rightarrow \{(N_0, [a]), (N_1, [a, a, a]), (N_2, [\,]), (N_3, [\,])\}$
$\Rightarrow \{(N_0, [a]), (N_1, [a, a]), (N_2, [a]), (N_3, [\,])\}$ $\Leftarrow k = 2$
$\Rightarrow \{(N_0, [a]), (N_1, [a, a]), (N_2, [a]), (N_3, [\,])\}$
$\Rightarrow \{(N_0, [a]), (N_1, [a, a]), (N_2, [a]), (N_3, [\,])\}$
$\Rightarrow \{(N_0, [a]), (N_1, [a]), (N_2, [a, a]), (N_3, [\,])\}$
$\Rightarrow \{(N_0, [a]), (N_1, [a]), (N_2, [a, a]), (N_3, [\,])\}$
$\Rightarrow \{(N_0, [a]), (N_1, [a]), (N_2, [\,]), (N_3, [a])\}$
$\Rightarrow \{(N_0, [a, a]), (N_1, [\,]), (N_2, [a]), (N_3, [\,])\}$

The output neuron N_3 spikes in steps 2 and 10
The number computed by this $\mathcal{L}_{SNP}$ program is
$3k + 2 = 8$
(same as in [Ionescu, Păun and Yokomori - 2006])

Two behavioraly equivalent $\mathcal{L}_{SNP}$ programs

$$\begin{aligned}
\rho_1 &= (D_1, x_1)\\
x_1 &= \text{send}(\langle a^{2k-1}\rangle, \{N_1\}) \parallel \text{send}(a, \{N_3\})\\
D_1 &= \ \text{neuron } N_0 \{ r_\epsilon \mid \{N_1, N_2, N_3\} \},\\
&\quad\ \text{neuron } N_1 \{ a^+/[a] \to a; 2 \mid \{N_2\} \},\\
&\quad\ \text{neuron } N_2 \{ [a^k] \to a; 1 \mid \{N_3\} \},\\
&\quad\ \text{neuron } N_3 \{ [a] \to a; 0 \mid \{N_0\} \}
\end{aligned}$$



[Ionescu, Păun and Yokomori – 2006]
SN P system Π_1

$$\begin{aligned}
\rho'_1 &= (D'_1, x'_1)\\
x'_1 &= \text{send}(\langle a^{2k-1}\rangle, \{N_1\}) \parallel \text{send}(a, \{N_3\})\\
D'_1 &= \ \text{neuron } N_0 \{ r_\epsilon \mid \{N_1, N_2, N_3\} \},\\
&\quad\ \text{neuron } N_1 \{ a^+/[a] \to \text{send}(a, \{N_2\}); 2 \mid \{N_2, N_3\} \},\\
&\quad\ \text{neuron } N_2 \{ [a^k] \to \text{send}(a, \{N_3\}); 1 \mid \{N_1, N_3\} \},\\
&\quad\ \text{neuron } N_3 \{ [a] \to a; 0 \mid \{N_0\} \}
\end{aligned}$$

Observables and final semantic domain [De Bakker and De Vink - 1996]

- Final semantic domain **P**, **Q** (linear time)

$$(p \in)\mathbf{P} = \mathcal{P}_{nco}(\mathbf{Q}) \qquad (q \in)\mathbf{Q} \cong \{\epsilon\} + (\Omega \times \frac{1}{2} \cdot \mathbf{Q})$$

- Set of observables Ω

$$(\omega \in)\Omega = \{\omega \mid \omega \in \mathcal{P}_{nfin}(Nname \times W),\ \nu(\omega)\}$$

- Nondeterministic choice operator $\oplus : (\mathbf{P} \times \mathbf{P}) \xrightarrow{1} \mathbf{P}$

$$p_1 \oplus p_2 = \{q \mid q \in p_1 \cup p_2, q \neq \epsilon\} \cup \{\epsilon \mid \epsilon \in p_1 \cap p_2\}$$

Remark

$\oplus$ is associative, commutative and idempotent

Computations and continuations [America and Rutten - 1989]

$$(\varphi \in)\mathbf{D} \cong \mathbf{K} \xrightarrow{1} \mathbf{K}$$

$$(\phi \in)\mathbf{Den} = \{d_0\} + \mathbf{D}$$

$$
\begin{array}{rcl}
(\kappa \in)\mathbf{K} & = & \Gamma \xrightarrow{1} \mathbf{P} \\
(\gamma \in)\Gamma & = & \{\!|\Sigma|\!\} \\
(\sigma \in)\Sigma & = & \mathbf{Open} + \mathbf{Closed} \\
\mathbf{Open} & = & \Xi \times W \\
\mathbf{Closed} & = & \Xi \times W \times \mathbb{N} \times W \times \frac{1}{2} \cdot \mathbf{D}
\end{array}
$$

- Continuations
- Configurations
- States (of neurons)

$\{\!|\Sigma|\!\} \stackrel{not.}{=} \Xi \times (Nname \to \Sigma)$ - Multisets / Bags (of states)

$[\gamma \mid N \mapsto \sigma]$ - Update (state of neuron N in γ with σ)

Continuation semantics for parallel composition

Definition (Semantics of $\|$ in continuation semantics)

We define $\|: (\mathbf{D} \times \mathbf{D}) \xrightarrow{1} \mathbf{D}$, $\lfloor : (\mathbf{D} \times \mathbf{D}) \xrightarrow{1} \mathbf{D}$ *by:*

$$\varphi_1 \| \varphi_2 = \lambda\kappa \,.\, \lambda\gamma \,.\, ((\varphi_1 \lfloor \varphi_2)(\kappa)(\gamma) \oplus (\varphi_2 \lfloor \varphi_1)(\kappa)(\gamma))$$

$$\varphi_1 \lfloor \varphi_2 = \varphi_1 \circ \varphi_2 \quad \text{('}\circ\text{' is function composition operator)}$$

Remarks

- $\|$ *and* $\lfloor$ *are well-defined and nonexpansive in both args*
- $\lfloor$ *is associative*
- $\|$ *is commutative (because* $\oplus$ *is commutative)*

Semantics of $\mathcal{L}_{SNP}$ statements

Definition (Denotational semantics $[\![\cdot]\!] : X \to Alpha \to \mathbf{D}$)

$$
\begin{array}{rcl}
[\![a]\!](\alpha) & = & \lambda\kappa \,.\, \lambda\gamma \,.\, \kappa(send(a, \alpha, \gamma)), \\
[\![\, \text{send}(y, \xi) \,]\!](\alpha) & = & [\![y]\!](\xi \Cap \alpha), \\
[\![x_1 \parallel x_2]\!](\alpha) & = & [\![x_1]\!](\alpha) \parallel [\![x_2]\!](\alpha).
\end{array}
$$

- $(\alpha \in) Alpha = Nname \times \Theta$
- $(\theta \in) \Theta = \{\text{all}\} \cup \Xi$
- $\Cap : (\Xi \times Alpha) \to Alpha$
  $\xi \Cap (N, \text{all}) = (N, \xi)$
  $\xi \Cap (N, \xi') = (N, \xi \cap \xi')$

The operation *send* : $(O \times Alpha \times \Gamma) \to \Gamma$

$$
\begin{aligned}
& send(a, (N, \text{all}), \gamma) = \\
& \qquad \text{let } \{N_1, \ldots, N_i\} = nbs(N, \gamma) \\
& \qquad \text{in } [\gamma \mid N_1 \mapsto add(a, \gamma(N_1)) \mid \cdots \mid N_i \mapsto add(a, \gamma(N_i))], \\
& send(a, (N, \xi), \gamma) = \\
& \qquad \text{let } \{N_1, \ldots, N_i\} = nbs(N, \gamma) \cap \xi \\
& \qquad \text{in } [\gamma \mid N_1 \mapsto add(a, \gamma(N_1)) \mid \cdots \mid N_i \mapsto add(a, \gamma(N_i))]
\end{aligned}
$$

$$
\begin{aligned}
& add(a, (\xi, w)) = (\xi, w \uplus [a]) \\
& add(a, (\xi, w, t, w_r, \varphi)) = (\xi, w, t, w_r, \varphi)
\end{aligned}
$$

Compositional reasoning with continuations

Proposition

(a) $[\![x_1]\!](\alpha_1) \parallel [\![x_2]\!](\alpha_2) = [\![x_1]\!](\alpha_1) \lfloor [\![x_2]\!](\alpha_2) = [\![x_2]\!](\alpha_2) \lfloor [\![x_1]\!](\alpha_1)$

(b) $[\![x_1 \parallel x_2]\!] = [\![x_2 \parallel x_1]\!]$

(c) $[\![x_1 \parallel (x_2 \parallel x_3)]\!] = [\![(x_1 \parallel x_2) \parallel x_3]\!]$

Proof.

Property (a) follows by induction on $size(x_1) + size(x_2)$; note that, in general, $\alpha_1 \neq \alpha_2$.
For property (c), let $x_1, x_2, x_3 \in X$, $\alpha \in Alpha$.

$$
\begin{aligned}
&[\![x_1 \parallel (x_2 \parallel x_3)]\!](\alpha) = [\![x_1]\!](\alpha) \parallel [\![x_2 \parallel x_3]\!](\alpha) \\
&= [\![x_1]\!](\alpha) \lfloor [\![x_2 \parallel x_3]\!](\alpha) = [\![x_1]\!](\alpha) \lfloor ([\![x_2]\!](\alpha) \parallel [\![x_3]\!](\alpha)) && \text{[Property (a)]} \\
&= [\![x_1]\!](\alpha) \lfloor ([\![x_2]\!](\alpha) \lfloor [\![x_3]\!](\alpha)) && [\ \lfloor\ \text{is associative}] \\
&= ([\![x_1]\!](\alpha) \lfloor [\![x_2]\!](\alpha)) \lfloor [\![x_3]\!](\alpha) \\
&= ([\![x_1]\!](\alpha) \parallel [\![x_2]\!](\alpha)) \lfloor [\![x_3]\!](\alpha) = [\![x_1 \parallel x_2]\!](\alpha) \lfloor [\![x_3]\!](\alpha) && \text{[Property (a)]} \\
&= [\![x_1 \parallel x_2]\!](\alpha) \parallel [\![x_3]\!](\alpha) = [\![(x_1 \parallel x_2) \parallel x_3]\!](\alpha)
\end{aligned}
$$

□

Semantics of $\mathcal{L}_{SNP}$ programs

Definition (Initial continuation κ_0)

Let $\Psi_{\mathbf{K}} : NDs \to \mathbf{K} \to \mathbf{K}$ *be given by*

$$\Psi_{\mathbf{K}}(D)(\kappa)(\gamma) = to_{\Omega}(\gamma) \cdot (\text{ if } halt_{NS}(\gamma, D) \text{ then } \{\epsilon\}$$
$$\text{else } \bigoplus\{\varphi(\kappa)(\gamma') \mid (\varphi, \gamma') \in sched(\gamma, D)\} \oplus$$
$$\bigoplus\{\kappa(\gamma') \mid (d_0, \gamma') \in sched(\gamma, D)\})$$

For any $D \in NDs$*, we define* $\kappa_0 \in \mathbf{K}$ *by* $\kappa_0 = \text{fix}(\Psi_{\mathbf{K}}(D))$.

Remark (Time is implicit in our denotational model)

- *In SN P systems functioning is synchronized*
- *A global clock is assumed; in our model the value of the clock is given by the number of* Ω *observable steps in each* $\mathbf{Q}$ *execution trace*
- $to_{\Omega} : \Gamma \to \Omega$ *produces an observable* $\omega \in \Omega$ *from a configuration* $\gamma \in \Gamma$

Semantics of $\mathcal{L}_{SNP}$ programs - scheduler mapping

$$\|: (\textbf{Den} \times \textbf{Den}) \xrightarrow{1} \textbf{Den},\ d_0 \parallel d_0 = d_0,\ d_0 \parallel \varphi = \varphi,\ \varphi \parallel d_0 = \varphi,\ \varphi_1 \parallel \varphi_2 = \varphi_1 \parallel \varphi_2$$

$$\begin{aligned}
&sched : (\Gamma \times NDs) \to \mathcal{P}_{co}(\textbf{Den} \times \Gamma) \\
&sched(\gamma, D) = \quad \text{let } \{N_0, \ldots, N_m\} = id(\gamma) \\
&\qquad \text{in } \{(\|_{i=0}^{m} \phi_i, [\gamma \mid N_0 \mapsto \sigma_0 \mid \cdots \mid N_m \mapsto \sigma_m]) \\
&\qquad \mid (\phi_0, \sigma_0) \in schedN(N_0, \gamma(N_0), D), \ldots, \\
&\qquad (\phi_m, \sigma_m) \in schedN(N_m, \gamma(N_m), D)\}
\end{aligned}$$

$$\begin{aligned}
&schedN : (Nname \times \Sigma \times NDs) \to \mathcal{P}_{co}(\textbf{Den} \times \Sigma) \\
&schedN(N, (\xi, w), D) = \text{ if } halt_N(N, (\xi, w), D) \text{ then } \{(d_0, (\xi, w))\} \\
&\qquad \text{else let } r = rules(D, N) \\
&\qquad \text{in } \{([\![x]\!](N, \text{all}), (\xi, w \setminus w_r)) \\
&\qquad \mid (E/w_r \to x; t) \in r, w \in L(E), w_r \subseteq w, t = 0\} \cup \\
&\qquad \{(d_0, (\xi, w, t-1, w \setminus w_r, [\![x]\!](N, \text{all}))) \\
&\qquad \mid (E/w_r \to x; t) \in r, w \in L(E), w_r \subseteq w, t > 0\} \cup \\
&\qquad \{(d_0, (\xi, [\,])) \mid (w_r \to \lambda) \in r, w_r = w\};
\end{aligned}$$

$$\begin{aligned}
&schedN(N, (\xi, w, t, w_r, \varphi), D) = \\
&\quad \text{if } t = 0 \text{ then } \{(\varphi, (\xi, w_r))\} \text{ else } \{(d_0, (\xi, w, t-1, w_r, \varphi))\}
\end{aligned}$$

Semantics of $\mathcal{L}_{SNP}$ programs: fixed-point (compositional) semantics

Proposition (κ_0 is well-defined (Banach))

$\Psi_{\mathbf{K}}(D) \in \mathbf{K} \xrightarrow{\frac{1}{2}} \mathbf{K}$ *($\Psi_{\mathbf{K}}(D)$ is a contraction), for any $D \in NDs$.*

Proof.

It suffices to prove the following (for any $D \in NDs, \kappa_1, \kappa_2 \in \mathbf{K}, \gamma \in \Gamma$):

$$d(\Psi_{\mathbf{K}}(D)(\kappa_1)(\gamma), \Psi_{\mathbf{K}}(D)(\kappa_2)(\gamma)) \leq \frac{1}{2} \cdot d(\kappa_1, \kappa_2)$$

We have

$$d(\Psi_{\mathbf{K}}(D)(\kappa_1)(\gamma), \Psi_{\mathbf{K}}(D)(\kappa_2)(\gamma))$$

["$to_\Omega(\gamma)$" - step in def. $\Psi_{\mathbf{K}}$, metric on $\mathbf{P}$, $\oplus$ is nonexpansive]

$$\leq \frac{1}{2} \cdot max\{max\{d(\varphi(\kappa_1)(\gamma'), \varphi(\kappa_2)(\gamma')) \mid (\varphi, \gamma') \in sched(\gamma, D)\}, max\{d(\kappa_1(\gamma'), \kappa_2(\gamma')) \mid (d_0, \gamma') \in sched(\gamma, D)\}\}$$

[$\varphi \in \mathbf{D}$, φ is nonexpansive]

$$\leq \frac{1}{2} \cdot d(\kappa_1, \kappa_2)$$

Semantics of $\mathcal{L}_{SNP}$ programs

Definition (Semantics of $\mathcal{L}_{SNP}$ programs)

We define $\mathcal{D}[\![\cdot]\!] : \mathcal{L}_{SNP} \to \mathbf{P}$ *for any* $\rho = (D, x) \in \mathcal{L}_{SNP}$ *by*

$$\mathcal{D}[\![\rho]\!] = \mathcal{D}[\![D, x]\!] = [\![x]\!](\alpha_0)(\kappa_0)(\gamma_0),$$

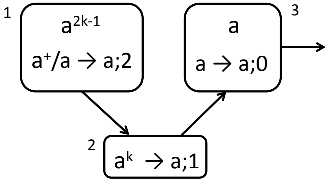
where $\kappa_0 = fix(\Psi_{\mathbf{K}}(D))$, $\gamma_o = init_{\Gamma}(D)$ *and* $\alpha_0 = (N_0, \mathrm{all})$.

- Haskell implementation provided in two variants:
  - `Lsnp.hs` - accurate implementation of $\mathcal{D}[\![\cdot]\!]$; can only run simple $\mathcal{L}_{SNP}$ programs like ρ_1 or ρ_1' (Π_1)
  - `Lsnp-fin.hs` - stops execution (prunes execution traces) after n steps; can run arbitrary $\mathcal{L}_{SNP}$ programs, including nonterminating and nondeterministic programs

Example: let $\rho_1 = (D_1, x_1)$, $\rho'_1 = (D'_1, x'_1)$ be as before

- ρ_1 implements deterministic SN P system Π_1

- $x_1 = \text{send}(\langle a^{2k-1}\rangle, \{N_1\}) \parallel \text{send}(a, \{N_3\})$
- $D_1 = $ neuron N_0 $\{ r_\epsilon \mid \{N_1, N_2, N_3\} \}$,
  neuron N_1 $\{ a^+/[a] \to a; 2 \mid \{N_2\} \}$,
  neuron N_2 $\{ [a^k] \to a; 1 \mid \{N_3\} \}$,
  neuron N_3 $\{ [a] \to a; 0 \mid \{N_0\} \}$
- $\omega_1 = \{(N_0, [\,]), (N_1, [a, a, a]), (N_2, [\,]), (N_3, [a])\}$
  $\omega_2 = \{(N_0, [a]), (N_1, [a, a, a]), (N_2, [\,]), (N_3, [\,])\}$
  $\omega_3 = \{(N_0, [a]), (N_1, [a, a, a]), (N_2, [\,]), (N_3, [\,])\}$
  $\omega_4 = \{(N_0, [a]), (N_1, [a, a]), (N_2, [a]), (N_3, [\,])\}$
  $\omega_5 = \{(N_0, [a]), (N_1, [a, a]), (N_2, [a]), (N_3, [\,])\}$
  $\omega_6 = \{(N_0, [a]), (N_1, [a, a]), (N_2, [a]), (N_3, [\,])\}$
  $\omega_7 = \{(N_0, [a]), (N_1, [a]), (N_2, [a, a]), (N_3, [\,])\}$
  $\omega_8 = \{(N_0, [a]), (N_1, [a]), (N_2, [a, a]), (N_3, [\,])\}$
  $\omega_9 = \{(N_0, [a]), (N_1, [a]), (N_2, [\,]), (N_3, [a])\}$
  $\omega_{10} = \{(N_0, [a, a]), (N_1, [\,]), (N_2, [a]), (N_3, [\,])\}$



[Ionescu, Păun and Yokomori – 2006]
SN P system Π_1

$$\mathcal{D}[\![\rho_1]\!] = [\![x_1]\!](\alpha_0)(\kappa_0)(\gamma_0) = \{\omega_1\omega_2\omega_3\omega_4\omega_5\omega_6\omega_7\omega_8\omega_9\omega_{10}\} = \mathcal{D}[\![\rho'_1]\!]$$

- Our Haskell interpreter `Lsnp-fin.hs` also runs nondeterministic SN P system Π_3 [Ionescu, Păun and Yokomori - 2006]; Π_3 computes all natural numbers (> 1)

Declarative protyping and denotational specification

```
type D = K -> K
data Den = D0 | D D

type K = Gamma -> P
type Gamma = Bag Sigma
data Sigma =
             Open Xi W
           | Closed Xi W Time W D
type P = [Q]
data Q = Q Omega Q | Epsilon

sem (Aspike a) alpha =
  \k gamma -> k (send a alpha gamma)
sem (Send y xi) alpha =
  sem y (xi `dintersect` alpha)
sem (Par x1 x2) alpha =
  (sem x1 alpha) `par` (sem x2 alpha)

k0 = fix (psiK nDs)
fix ::  (a -> a) -> a
fix f = f (fix f)
```

$$\mathbf{D} \cong \mathbf{K} \xrightarrow{1} \mathbf{K}$$
$$\mathbf{Den} = \{d_0\} + \mathbf{D}$$

$$\mathbf{K} = \Gamma \xrightarrow{1} \mathbf{P}$$
$$\Gamma = \{\!|\Sigma|\!\}$$
$$\Sigma = \mathbf{Open} + \mathbf{Closed}$$
$$\mathbf{Open} = \Xi \times W$$
$$\mathbf{Closed} = \Xi \times W \times \mathbb{N} \times W \times \tfrac{1}{2} \cdot \mathbf{D}$$
$$\mathbf{P} = \mathcal{P}_{nco}(\mathbf{Q})$$
$$\mathbf{Q} \cong \{\epsilon\} + (\Omega \times \tfrac{1}{2}\mathbf{Q})$$

$$[\![a]\!](\alpha) = \lambda\kappa\,.\,\lambda\gamma\,.\,\kappa(send(a,\alpha,\gamma))$$
$$[\![\,\text{send}(y,\xi)\,]\!](\alpha) = [\![y]\!](\xi \Cap \alpha)$$
$$[\![x_1 \parallel x_2]\!](\alpha) = [\![x_1]\!](\alpha) \parallel [\![x_2]\!](\alpha)$$

$$\kappa_0 = \text{fix}(\Psi_{\mathbf{K}}(D))$$

Conclusion and future research

- We offer a denotational semantics $[\![\cdot]\!]$ for an experimental concurrent language $\mathcal{L}_{SNP}$ inspired by the SN P systems
  - $[\![\cdot]\!]$ is designed with metric domains and continuations
  - Accurately describes the behavior of SN P systems
    - Including time delays and synchronized functioning
  - Offers support for reasoning about the behavior of $\mathcal{L}_{SNP}$
  - Haskell implementation available from

    `http://ftp.utcluj.ro/pub/users/gc/eneia/cmc19`

- In the future we could
  - Study the abstractness of denotational vs operational semantics of SN P systems [Ciobanu & Todoran - 2018]
  - Develop language support and formal verification tools for SN P systems extended with: dynamical structure, quantitative aspects (stochastic/fuzzy), compositionality

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