Membrane Computing with Water

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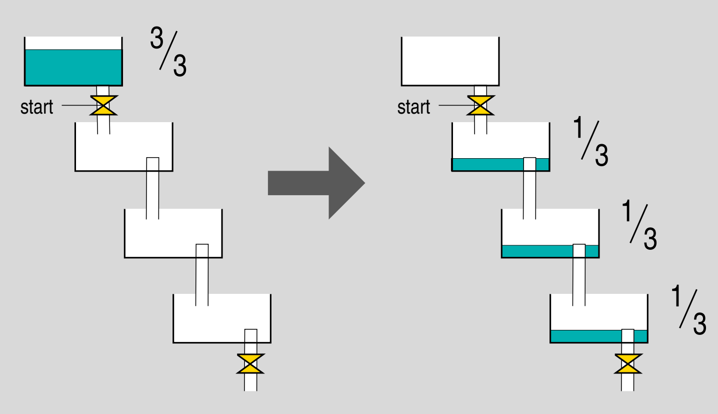
Water in Its Liquid Form . . .

. . . an ideal *medium* for doing and for *visualisation* of *information processing*

Ancient Water Cascade ≈ 2000 Years Ago



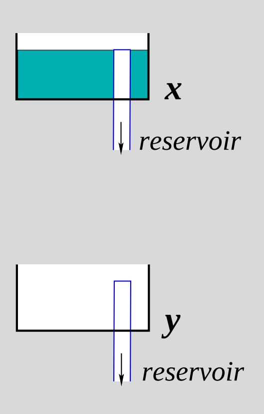
separates *initial pool of water* into nearly equal *portions*

K. Mahatantila et al. Spatial and temporal changes of hydrogeochemistry in ancient tank cascade systems in Sri Lanka: evidence for a constructed wetland. *Water and Environment Journal* **22**:17-24, 2008

Idea of a Water Tank System

Finite number of water tanks

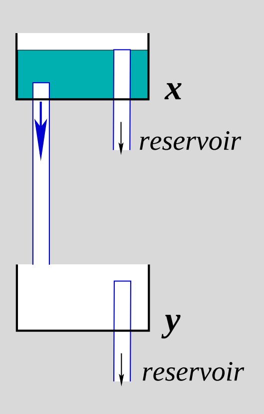
- *water tank* with *identifier* and *initial volume of water*
- stores an amount of water up to a maximum volume
- filled or emptied to conduct computations
- equipped with *overflow pipe*
- special water tank called *reservoir* for supply and for collection of excessive water
- counterpart of a *membrane*



Idea of a Water Tank System

Finite number of pipes

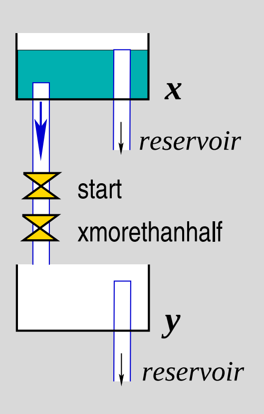
- *pipe* connects *two water tanks* for directed transport of water
- transported volume of water within one time step when opened given by *volumetric flow rate* $\dot{v}$
- *entrance* might be placed *at an arbitrary filling level* of its supply tank in a fixed position
- allowed to be equipped with one or more freely configurable *valves*



Idea of a Water Tank System

Finite number of valves

- a *valve* either *fully opens* or *fully closes* its hosting *pipe* dependent on *filling level* in a dedicated *water tank*
- freely configurable (selection of *water tank*, open/close iff *filling level* exceeds/reaches/is below of a *threshold*)
- in addition, *start valves* simultaneously open hosting pipes at a configurable point in time to initiate computation
- arbitrary number of *valves* per *pipe* in line, all have to be opened to enable *water transport*



Definition of a Water Tank System (WTS)

Let $\mathbb{Q}_+$ be the set of non-negative rational numbers.
A water tank system $\Pi_\sim$ is a construct

$$\Pi_\sim = (\mathcal{W}, A, P, t_{\text{start}}, t_{\text{end}}) \quad \text{with}$$

$\mathcal{W} : \Sigma \longrightarrow \mathbb{Q}_+ \cup \{\infty\}$ .. finite fractional multiset of *water tanks*

- alphabet Σ of water tank identifiers
- $\mathcal{W}(w)$ specifies initial volume of water in tank $w \in \Sigma$
- $\mathcal{W}(w) = \infty$ holds for the *reservoir*.
- quantity $V_w(t) \in \mathbb{Q}_+$ captures volume of water in tank w at time $t \in \mathbb{N}$, $V_w(0) = \mathcal{W}(w)$
- values $V_w(t) \quad \forall w \in \Sigma$ represent *configuration* of the system at time $t \in \mathbb{N}$

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$A \subset \Sigma \times \mathbb{Q}_+ \times \{0,1\}$ ....finite set of *valves* (actors)

- valve $\mathrm{a} : \Sigma \times \mathbb{Q}_+ \longrightarrow \{0,1\}$ is a boolean function of the forms $\mathrm{a}(w, \Theta) = \begin{cases} 1 \text{ (open)} & \text{iff } V_w(t) \square \Theta \\ 0 \text{ (close)} & \text{otherwise} \end{cases}$ with $\square = > \mid \geq \mid < \mid \leq$, water tank w, and threshold Θ.
- Start valve: $\mathrm{a}_{\text{start}} = \begin{cases} 1 \text{ (open)} & \text{iff } t \geq t_{\text{start}} \\ 0 \text{ (close)} & \textit{otherwise} \end{cases}$

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$P \subset \Sigma \times \Sigma \times A \times \mathbb{Q}_+ \times \mathbb{Q}_+$ .... finite set of *pipes*

- *pipe* $p = (s, w, B, \dot{v}, \Theta_s)$ with *valves* $B \subseteq A$ enables directed transport of the volume portion $\dot{v}$ of water from s to w within one time step if and only if
  - all valves placed at the pipe are opened $\forall \mathrm{b} \in B \ : \ (\mathrm{b} = 1)$
  - start valve if placed is open $\mathrm{b}_{\text{start}} = 1$
  - supply tank contains portion of water $V_s(t) \geq \dot{v}$
  - supply tank has minimum filling level $V_s(t) \geq \Theta_s$

Definition of a Water Tank System (WTS)

Let $\mathbb{Q}_+$ be the set of non-negative rational numbers.
A water tank system $\Pi_\sim$ is a construct

$$\Pi_\sim = (\mathcal{W}, A, P, t_{\text{start}}, t_{\text{end}}) \quad \text{with}$$

$t_{\text{start}} \in \mathbb{N}$ ....point in time to start

$t_{\text{end}} \in \mathbb{N}$ with $t_{\text{end}} > t_{\text{start}}$ ....point in time to terminate

Systems behaviour

$t := 0$

while $t < t_{\text{end}}$

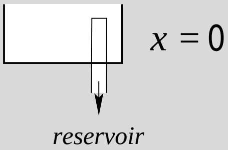
for all $(s, w, B, \dot{v}, \Theta_s) \in P$

if $\left(\prod_{b \in B} b(x, u) = 1 \text{ with } x \in \Sigma \text{ and } u \in \mathbb{Q}_+\right) \wedge (b_{\text{start}} = 1) \wedge$
$(V_s(t) \geq \dot{v}) \wedge (V_s(t) \geq \Theta_s)$

$V_s(t+1) := V_s(t) - \dot{v}$

$V_w(t+1) := V_w(t) + \dot{v}$

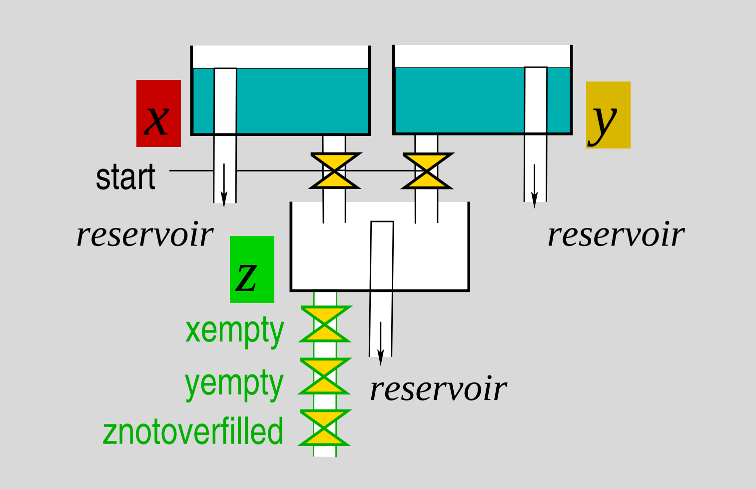
$t := t + 1$



Examples in Binary Mode

OR Gate – System's Topology

$z = x \vee y$



output water tank *z* can act as input for subsequent logic gates

OR Gate – Formal Definition of WTS

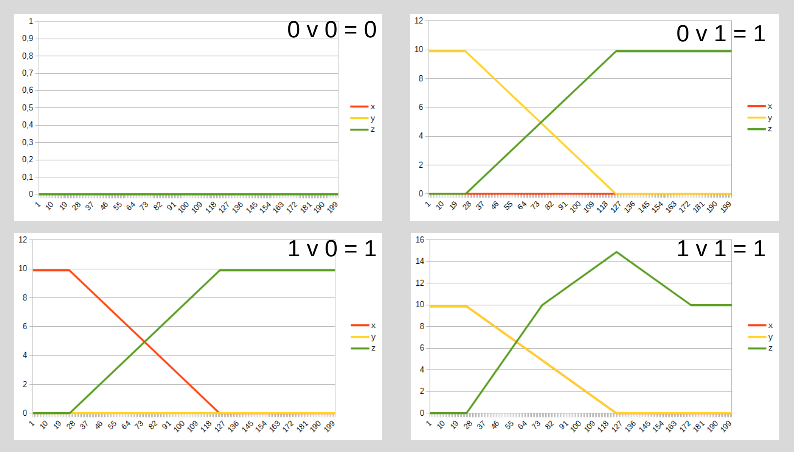
$$z = x \vee y$$

$$\begin{aligned}
\Pi_{\mathrm{OR}} &= (\mathcal{W}, A, P, 25, 200) \text{ with} \\
\mathcal{W} &= \{(x, 10), (y, 10), (z, 0), (reservoir, +\infty)\} \\
A &= \left\{ \left(\text{start} = \begin{cases} 1 & \text{iff } t \geq 25 \\ 0 & \text{otherwise} \end{cases} \right) \right\} \\
P &= \{(x, z, \{\text{start}\}, 0.1, 0), (x, reservoir, \emptyset, 0.1, 10), \\
&\quad (y, z, \{\text{start}\}, 0.1, 0), (y, reservoir, \emptyset, 0.1, 10), \\
&\quad (z, reservoir, \emptyset, 0.1, 10)\}
\end{aligned}$$

- volume of 10 units encodes logical value “1”, empty tank represents “0”
- common volumetric flow rate $\dot{v} = 0.1$ assigned to all pipes
- output z valid as soon as both input tanks x, y empty, and tank z is not overfilled any more

OR Gate – Simulation Results

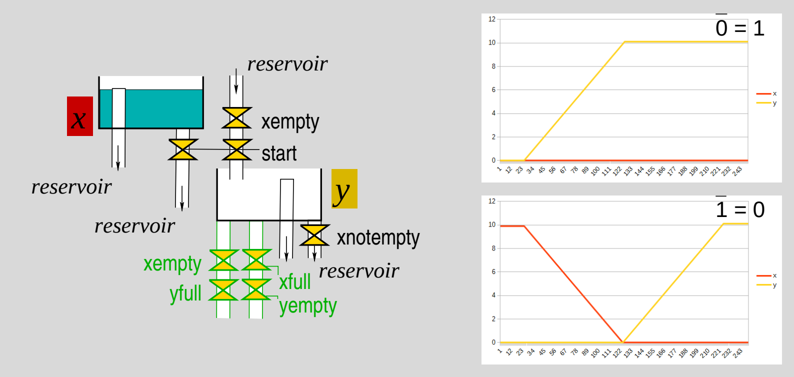
$z = x \vee y$



courses of $V_x(t)$, $V_y(t)$, and $V_z(t)$ over time

NOT Gate (Inverter)

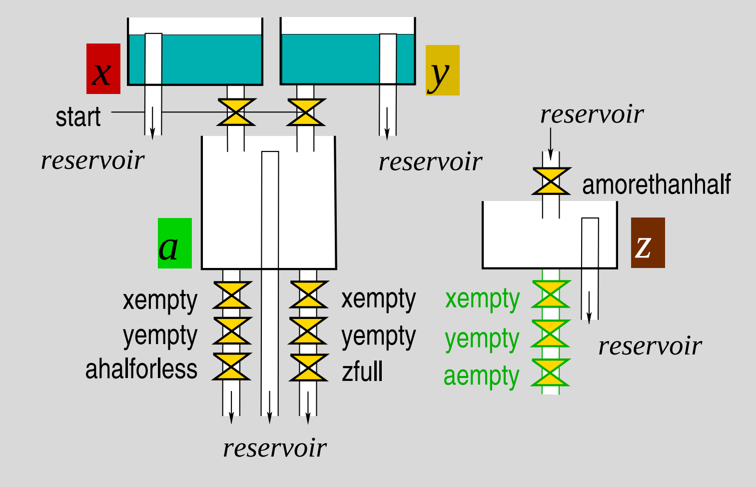
$$y = \overline{x}$$



- two valves indicate empty and non-empty input tank x
- output water tank y can act as input for downstream gates
- courses of $V_x(t)$ and $V_y(t)$ over time depicted

AND Gate – System's Topology

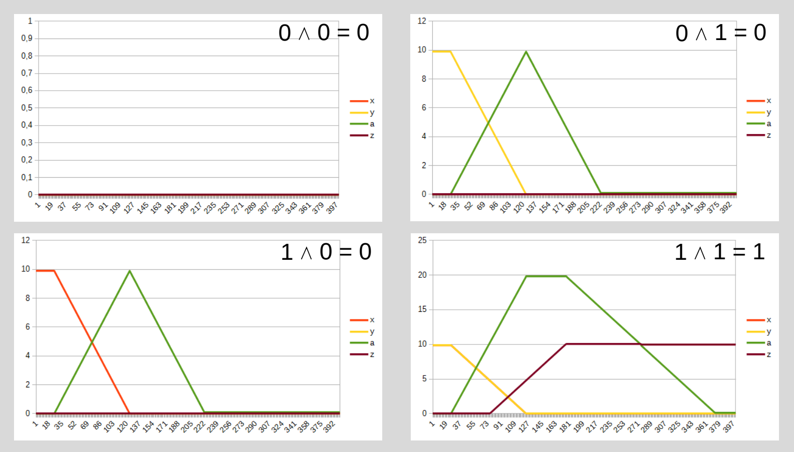
$z = x \wedge y$



output water tank z can act as input for subsequent logic gates

AND Gate – Simulation Results

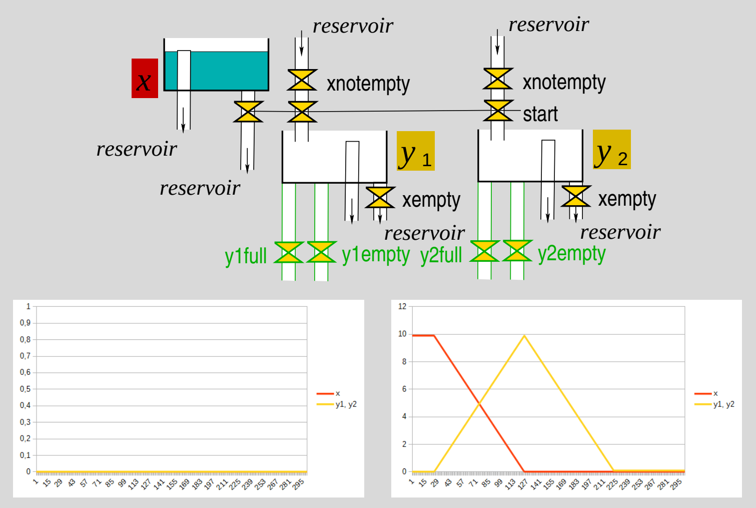
$$z = x \wedge y$$



courses of $V_x(t)$, $V_y(t)$, $V_a(t)$, and $V_z(t)$ over time

Bit Duplicator

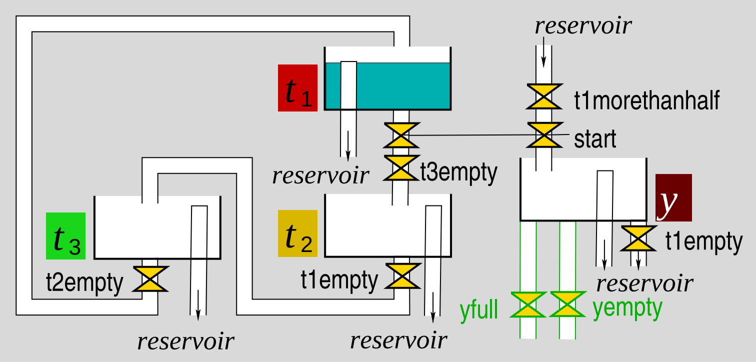
$y_1 = x, \quad y_2 = x$ for multiple usage of a binary signal x



output water tanks y_1, y_2 can act as input for subsequent logic gates

Ring Oscillator for Generation of a Clock Signal

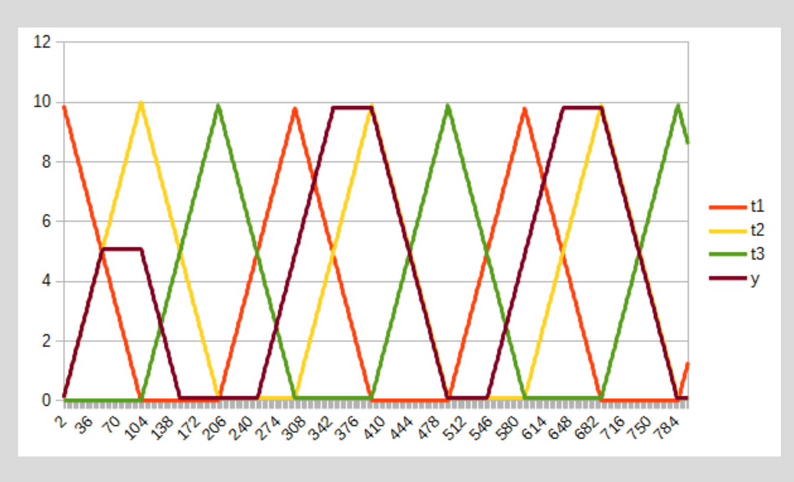
in output tank *y*



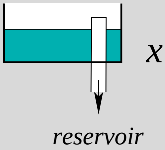
- cyclic scheme composed of at least three tanks (t_1, t_2, t_3)
- bit duplicaton of t_1 provides output in tank y
- self-sustained oscillation by permanent successive rotation of a water portion alongside the cycle after start

Ring Oscillator – Simulation Results

clock signal after transient phase in output tank *y*



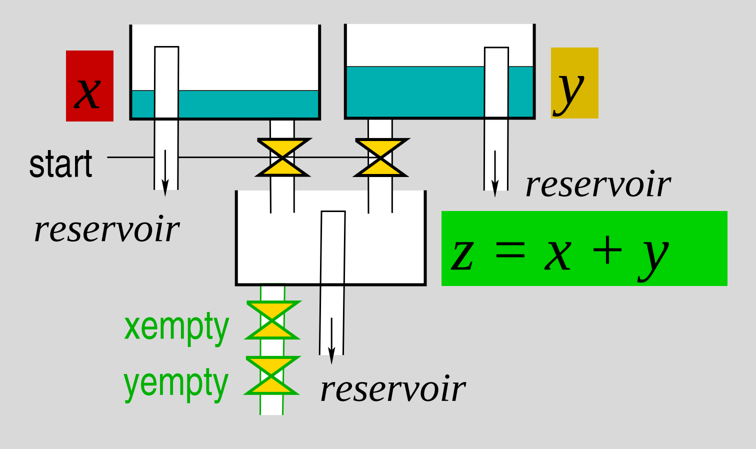
courses of $V_{t_1}(t)$, $V_{t_2}(t)$, $V_{t_3}(t)$, and $V_y(t)$ over time



Examples in Analogue Mode

Addition – System's Topology

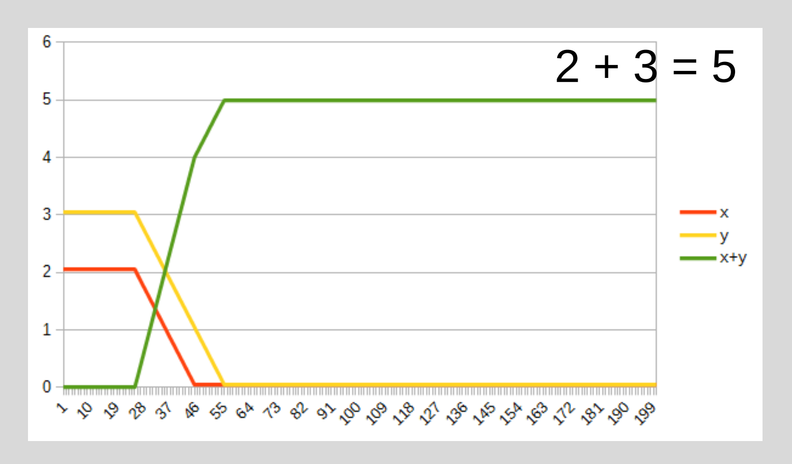
$z = x + y$



output tank z can act as input for further arithmetic operations

Addition – Simulation Results

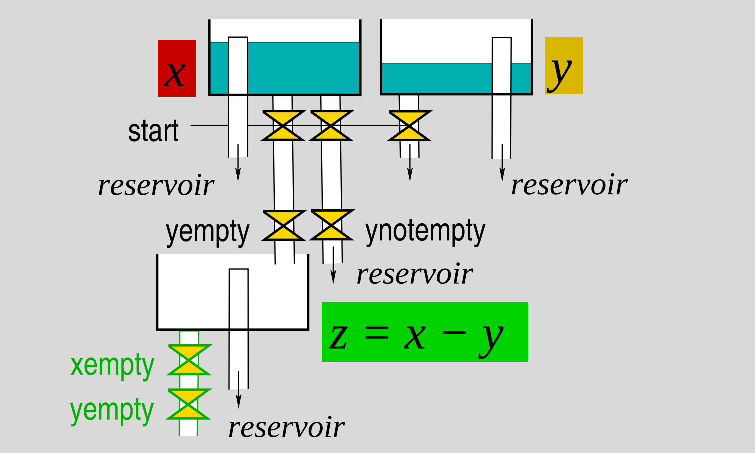
$z = x + y$



courses of $V_x(t)$, $V_y(t)$, and $V_{z=x+y}(t)$ over time

Non-negative Subtraction – System's Topology

$z = x \dot{-} y$



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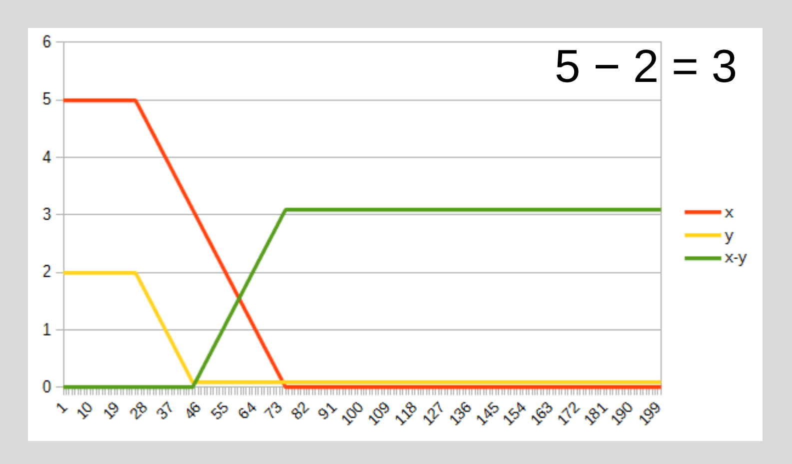
Non-negative Subtraction – Simulation Results

$z = x \dot{-} y$



courses of $V_x(t)$, $V_y(t)$, and $V_{z=x \dot{-} y}(t)$ over time

Integer Division – Idea

$$z = x/y$$

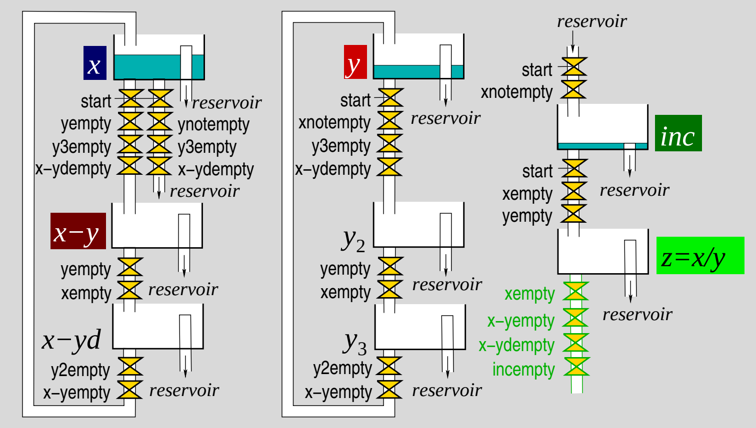
- employ *recursive scheme*

$$\left\lceil \frac{x}{y} \right\rceil = \begin{cases} 0 & \text{iff } x < y \;\wedge\; x = 0 \\ 1 & \text{iff } x < y \;\wedge\; x > 0 \\ \left\lceil \frac{x-y}{y} \right\rceil + 1 & \text{otherwise} \end{cases}$$

  in which $x, y \in \mathbb{N}$ and $y \geq 1$
- adapt the *ring oscillator* and combine it with a *subtractor*
- in each *cycle*, initial water volume x is *diminished* by the portion y and its result $x \dot{-} y$ iterated as x in next cycle
- a *second cycle* iteratively makes available the water portion y for consecutive subtraction in the first cycle
- both cycles *self-synchronise* to each other
- each cycle *accumulatively* provides an amount of 1 unit of water in tank *inc* *collected* for the *final division output*

Integer Division – System's Topology

$z = x/y$



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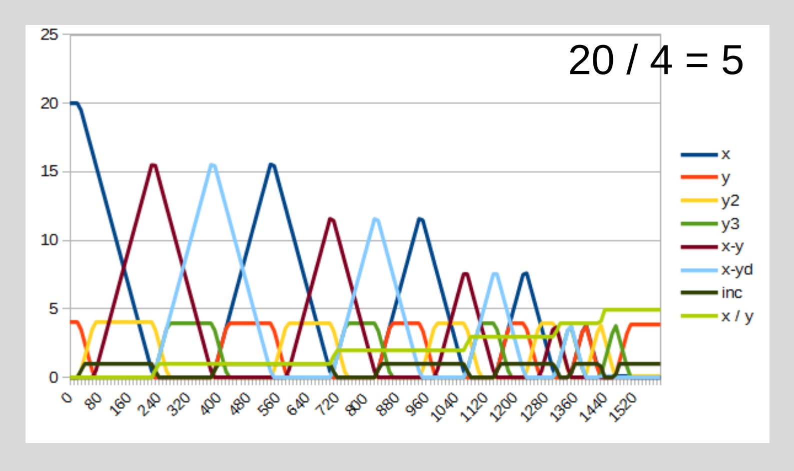
Integer Division – Simulation Results

$z = x/y$



courses of $V_x(t)$, $V_y(t)$, ..., and $V_{z=x/y}(t)$ over time

Integer Multiplication – Idea

$$z = x \cdot y$$

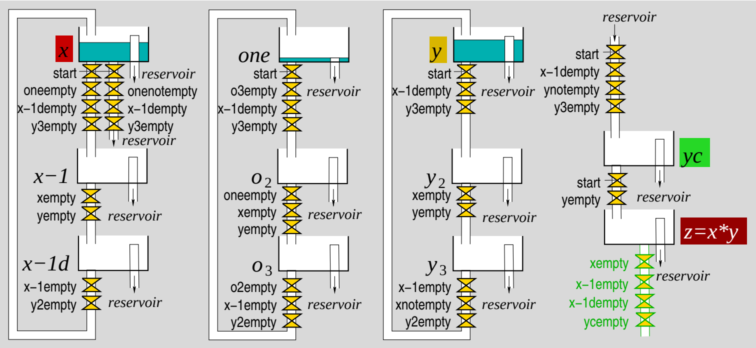
- employ *recursive scheme*

$$x \cdot y = \begin{cases} y & \text{iff } x = 1 \\ (x-1) \cdot y + y & \text{otherwise} \end{cases}$$

  in which $x, y \in \mathbb{N}$ and $x \geq 1$
- adapt the *ring oscillator* and combine it with a *subtractor*
- in each *cycle*, initial water volume x is *diminished* by the portion 1 and its result $x \dot{-} 1$ iterated as x in next cycle
- a *second cycle* iteratively makes available the water portion 1 for consecutive subtraction in the first cycle
- *third cycle* needed in order to iteratively provide water portion of y units for *accumulation*
- all cycles *self-synchronise* to each other
- accumulated portions y successively collected to form *final multiplication output*

Integer Multiplication – System's Topology

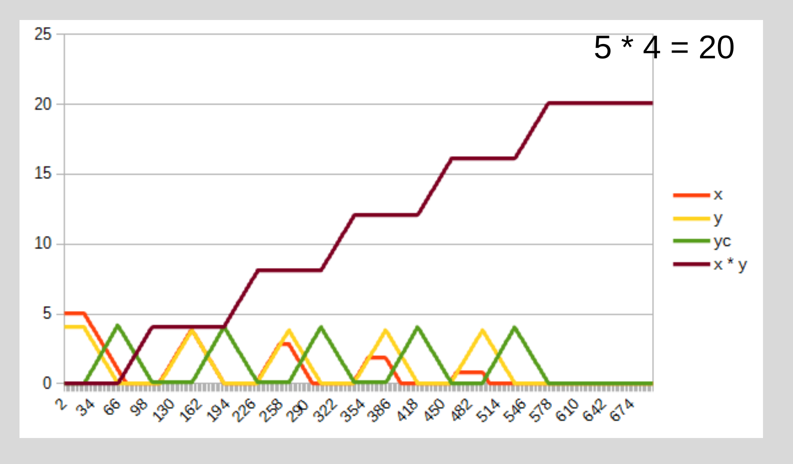
$$z = x \cdot y$$



output tank z can act as input for further arithmetic operations

Integer Multiplication – Simulation Results

$z = x \cdot y$



courses of $V_x(t)$, $V_y(t)$, ..., and $V_{z=x \cdot y}(t)$ over time

Take Home Message

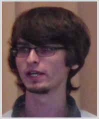
Water tank systems open an inspirational and interesting concept for modelling and visualisation of computational processes. A crucial feature of a water tank system lies in its ability to function without any central or external control.

Conclusions

- Combinable logic gates together with clock by ring oscillator enable construction of a water-based Turing-complete model for computation.
- Water-based arithmetics in analogue mode inspires algorithmic concepts and sensibilises for challenges of analogue computers.
- Further research dedicated to application for modelling and exploration of biological control loops incorporating blood circulation and affectation throughout organs.

Thank you for your attention.

Thomas Hinze (Jena)



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