Parallel Computing With Water

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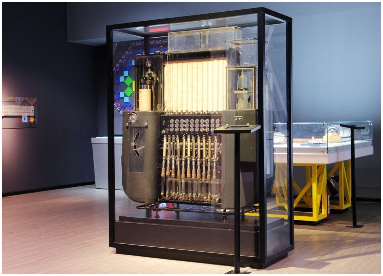
Outline of presentation

The power of water

Coromandel, New Zealand

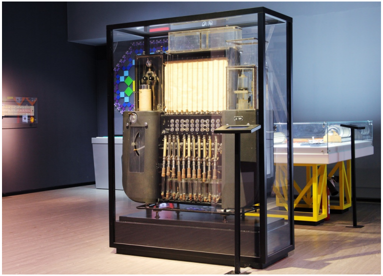
The magical sound,
of the cascading water,
natural beauty

Water integrator



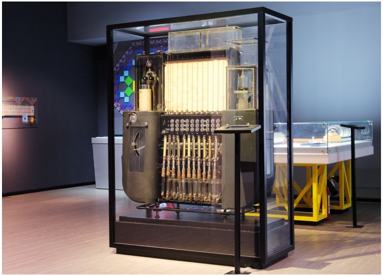
- First model built in 1936, in USSR; modular model in 1941, standard unified units in 1949-1955

Water integrator



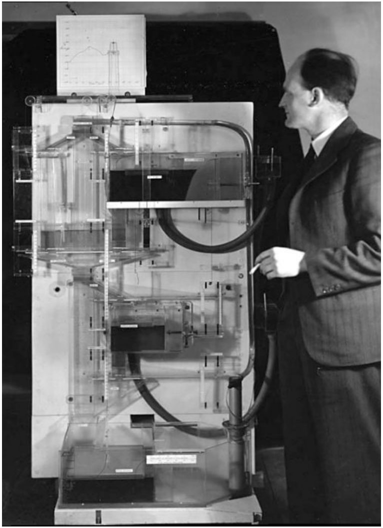
- First model built in 1936, in USSR; modular model in 1941, standard unified units in 1949-1955
- Used to solve inhomogeneous differential equations with applications such as: solving construction issues in the sands of Central Asia and in permafrost and in studying the temperature regime of the Antarctic ice sheet

Water integrator



- First model built in 1936, in USSR; modular model in 1941, standard unified units in 1949-1955
- Used to solve inhomogeneous differential equations with applications such as: solving construction issues in the sands of Central Asia and in permafrost and in studying the temperature regime of the Antarctic ice sheet
- Only surpassed by digital computers in the 1980's.

MONIAC (Monetary National Income Analogue Computer)



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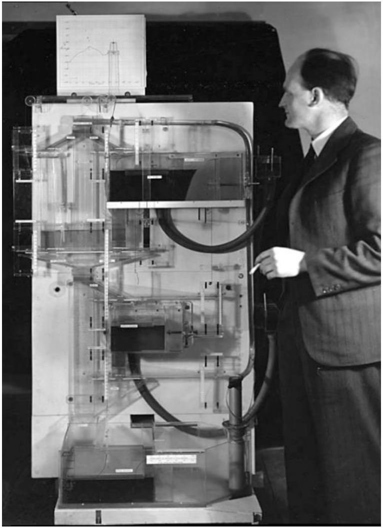

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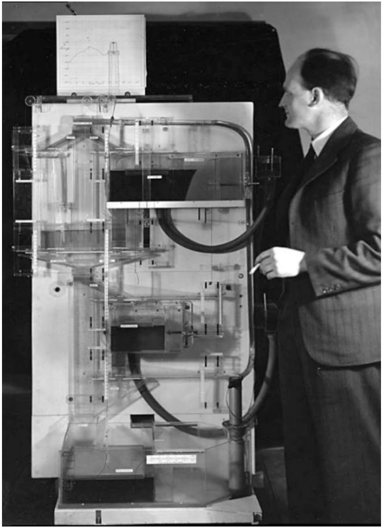
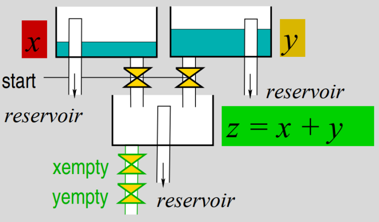


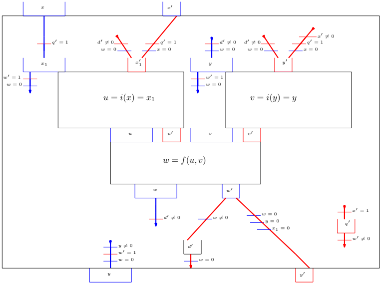
- MONIAC (Monetary National Income Analogue Computer) also known as the Phillips Hydraulic Computer and the Financephalograph
- First built in 1949 by New Zealand economist Bill Phillips to model the UK economy.
- Built as a teaching aid it was discovered that it was also an effective economic simulator.

Previous work[1]



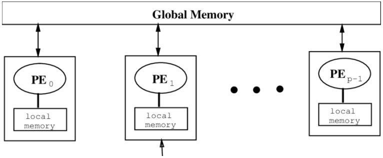
- No centre of control.
- Water flows if and only if all valves on a pipe are open.
- Water flows between tanks concurrently.

[1] Thomas Hinze et al. "Membrane computing with water". In: *J. Membr. Comput.* 2.2 (2020), pp. 121–136.



- Proved Turing complete via μ-recursive functions.
- Control tanks on input and output.

Problems



- Design a "practical" computational device.
- Show how to utilise the parallelism of the model via construction of Parallel Random Access Machine (PRAM).

RAM

Program p : GCD(a,b)

$r_3 \leftarrow r_1 \ominus r_2$	3	3	1	2
TRA 6 $r_3 > 0$	6	6	3	
$r_3 \leftarrow r_2 \ominus r_1$	3	3	2	1
TRA 8 $r_3 > 0$	6	8	3	
TRA 10 $r_1 > 0$	6	10	1	
$r_1 \leftarrow r_1 \ominus r_2$	3	1	1	2
TRA 1 $r_1 > 0$	6	1	1	
$r_2 \leftarrow r_2 \ominus r_1$	3	2	2	1
TRA 1 $r_2 > 0$	6	2	1	

- GCD(a,b): program of $m = 9$ lines
- Sequence of registers $r_1 = a$ (then result), $r_2 = b$, $r_3 = 0$

Op codes

1. $r_i \leftarrow C$: assign a constant value C to register i.
2. $r_i \leftarrow r_j \oplus r_k$: add the value of two registers j and k and assign to register i.
3. $r_i \leftarrow r_j \ominus r_k$: subtract from register j the value stored in k and assign to register i.
4. $r_i \leftarrow r_{r_j}$: get the value y from register j, then get the value from register y and assign to register i
5. $r_{r_i} \leftarrow r_j$: get the value y from register j, then get the value x from register i and assign y to register x.
6. TRA m $r_i > 0$: go to program line m (control transferred to line m of the program) if r_i greater than 0, otherwise go to the next line.

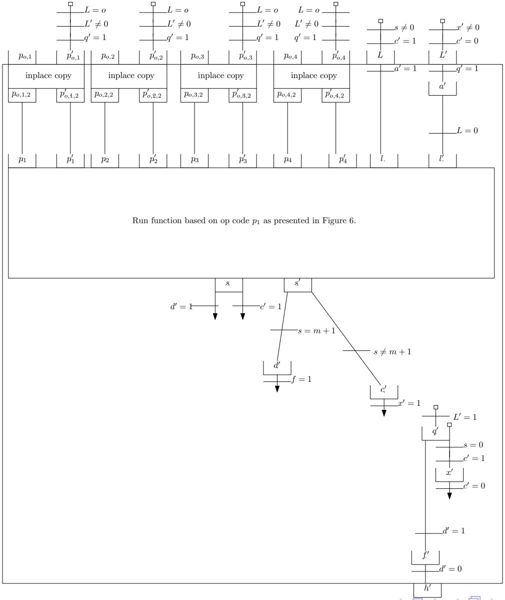
Op codes

Table: Operations and there corresponding opcodes.

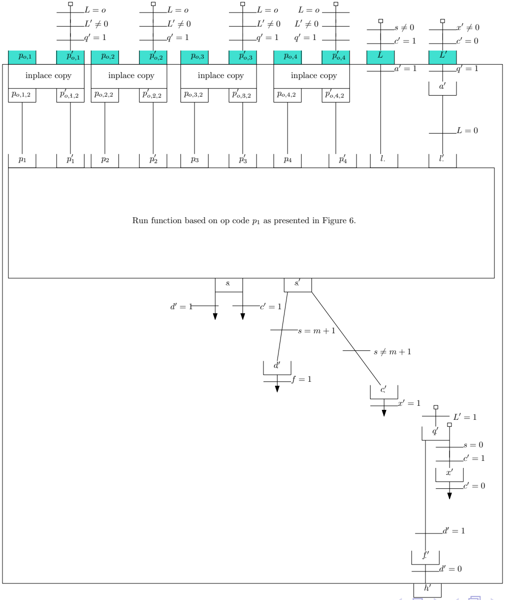
Name	Operation	Opcode
const	$r_i \leftarrow C$	$1\ i\ C$
add	$r_i \leftarrow r_j \oplus r_k$	$2\ i\ j\ k$
sub	$r_i \leftarrow r_j \ominus r_k$	$3\ i\ j\ k$
indr	$r_i \leftarrow r_{r_j}$	$4\ i\ j$
indw	$r_{r_i} \leftarrow r_j$	$5\ i\ j$
tra	TRA m $r_i > 0$	$6\ m\ i$

Outer loop

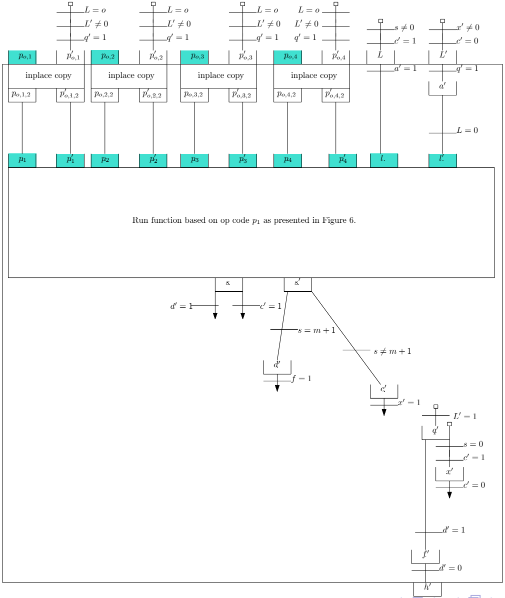
```
Procedure RAM(p, L)//p = [p_{1,1}, p_{1,2}, p_{1,3}, p_{1,4}, p_{2,1}, p_{2,2}, ..., p_{m,4}]
    p_1 ← p[4 ∗ (L − 1)]; p_2 ← p[4 ∗ (L − 1) + 1]
    p_3 ← p[4 ∗ (L − 1) + 2]; p_4 ← p[4 ∗ (L − 1) + 3]
    s ← run_op(p_1, p_2, p_3, p_4, L)
    if s ≠ m + 1 then
        RAM(p, s)
    halt
```


$L = o$
$L' \neq 0$
$q' = 1$
$s \neq 0$
$c' = 1$
$x' \neq 0$
$c' = 0$
$p_{o,1}$ $p'_{o,1}$ $p_{o,2}$ $p'_{o,2}$ $p_{o,3}$ $p'_{o,3}$ $p_{o,4}$ $p'_{o,4}$ L L'
inplace copy
$a' = 1$
$q' = 1$
a'
$p_{o,1,2}$ $p'_{o,1,2}$ $p_{o,2,2}$ $p'_{o,2,2}$ $p_{o,3,2}$ $p'_{o,3,2}$ $p_{o,4,2}$ $p'_{o,4,2}$
$L = 0$
p_1 p'_1 p_2 p'_2 p_3 p'_3 p_4 p'_4 $l.$ $l'.$
Run function based on op code p_1 as presented in Figure 6.
s s'
$d' = 1$
$c' = 1$
$s = m + 1$
$s \neq m + 1$
d'
$f = 1$
c'
$x' = 1$
$L' = 1$
q'
$s = 0$
$c' = 1$
x'
$c' = 0$
$d' = 1$
f'
$d' = 0$
h'

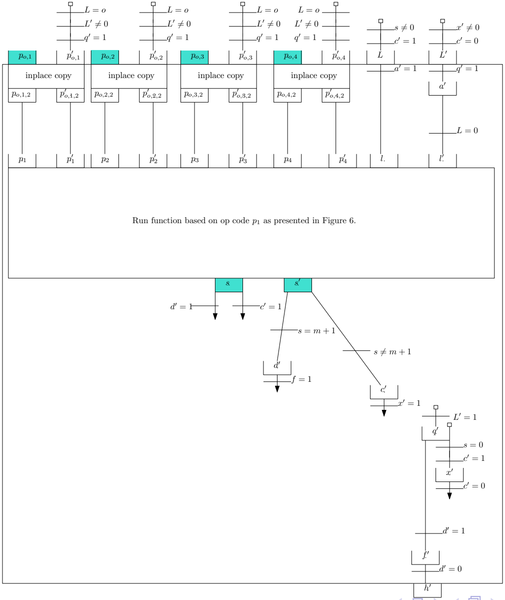
Outer loop Step 1



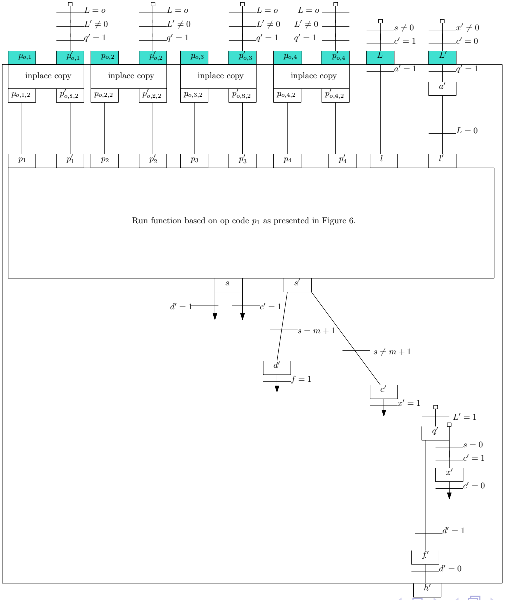
Outer loop Step 2

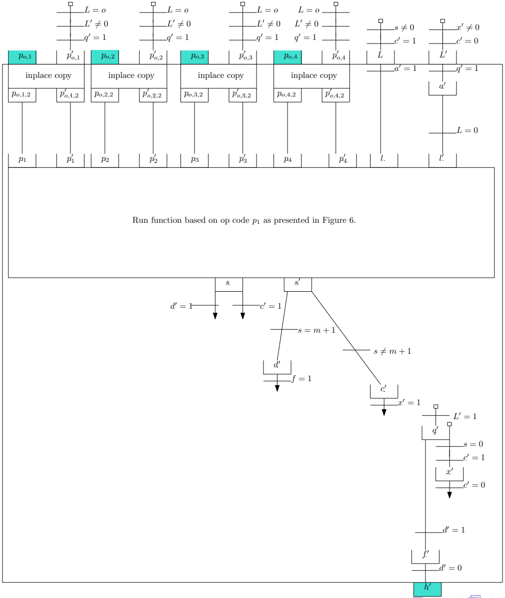


Outer loop Step 3



Outer loop Step 4(i)




$L = o$
$L' \neq 0$
$q' = 1$
$p_{o,1}$
$p'_{o,1}$
$p_{o,2}$
$p'_{o,2}$
$p_{o,3}$
$p'_{o,3}$
$p_{o,4}$
$p'_{o,4}$
inplace copy
$p_{o,1,2}$
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$p'_{o,2,2}$
$p_{o,3,2}$
$p'_{o,3,2}$
$p_{o,4,2}$
$p'_{o,4,2}$
$s \neq 0$
$c' = 1$
L
$a' = 1$
$x' \neq 0$
$c' = 0$
L'
$q' = 1$
a'
$L = 0$
p_1
p'_1
p_2
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p'_4
$l.$
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Run function based on op code p_1 as presented in Figure 6.
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s'
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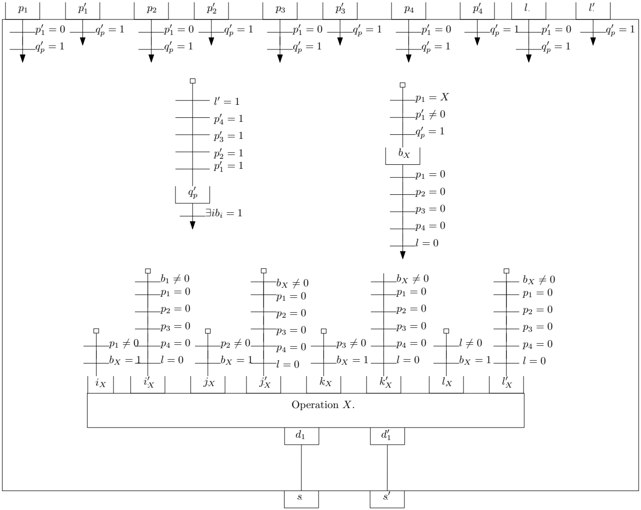
$s = m + 1$
$s \neq m + 1$
d'
$f = 1$
c'
$x' = 1$
$L' = 1$
q'
$s = 0$
$c' = 1$
x'
$c' = 0$
$d' = 1$
f'
$d' = 0$
h'

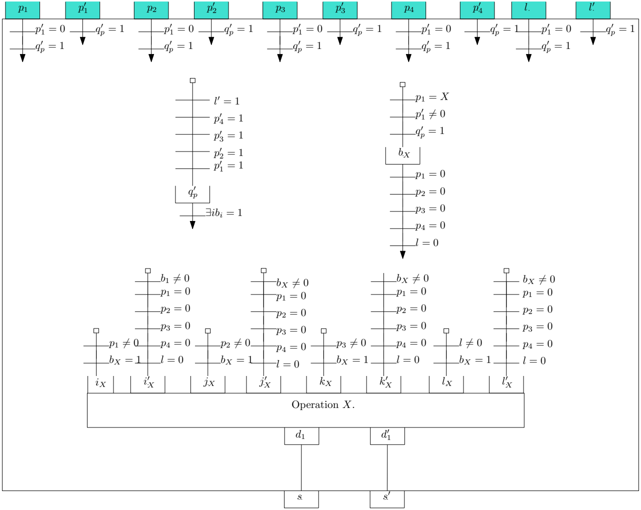
Operation selection

```
Procedure run_op(p1, p2, p3, p4, l)
    switch p1
        case 1
            return const(p2, p3, l)
        case 2
            return add(p2, p3, p4, l)
        case 3
            return sub(p2, p3, p4, l)
        case 4
            return indr(p2, p3, l)
        case 5
            return indw(p2, p3, l)
        case 6
            return tra(p2, p3, l)
```

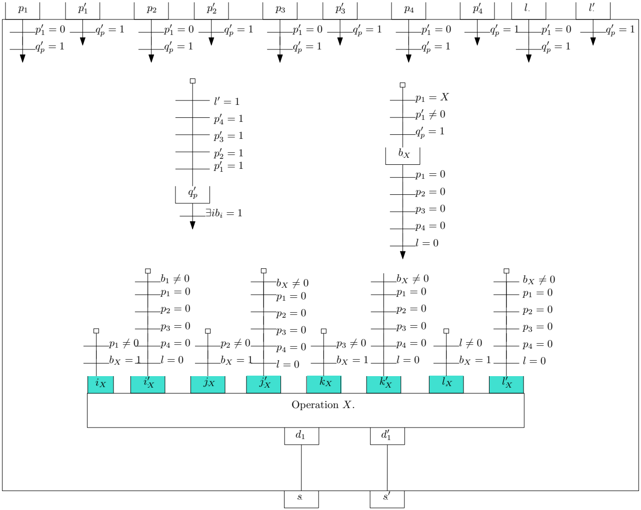
Operation selection



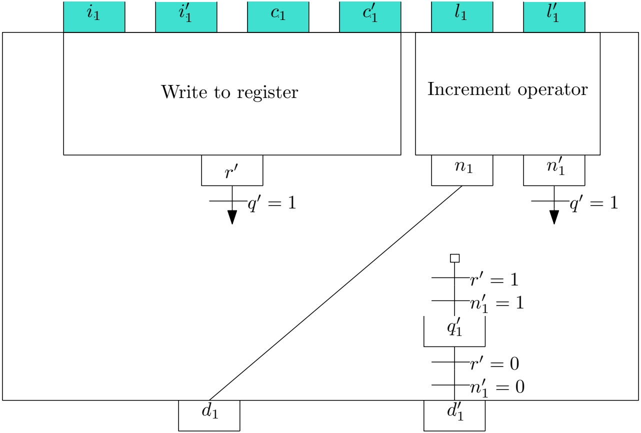
Operation selection step 1



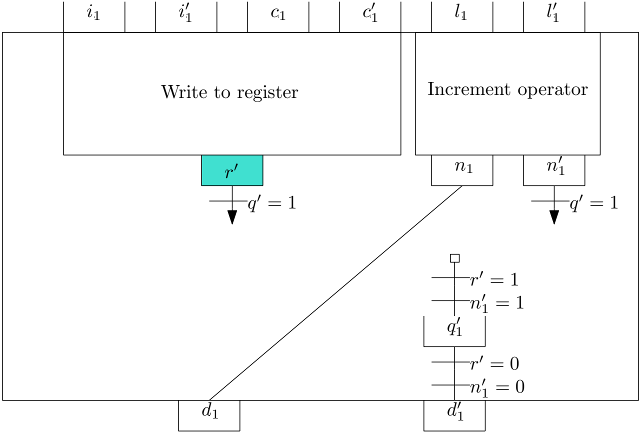
Operation selection step 2



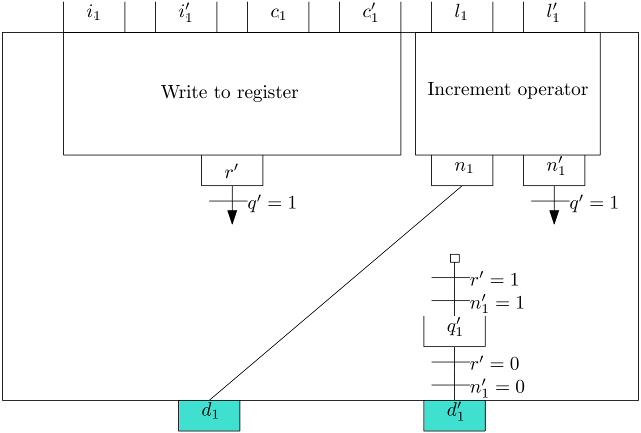
Constant step 1



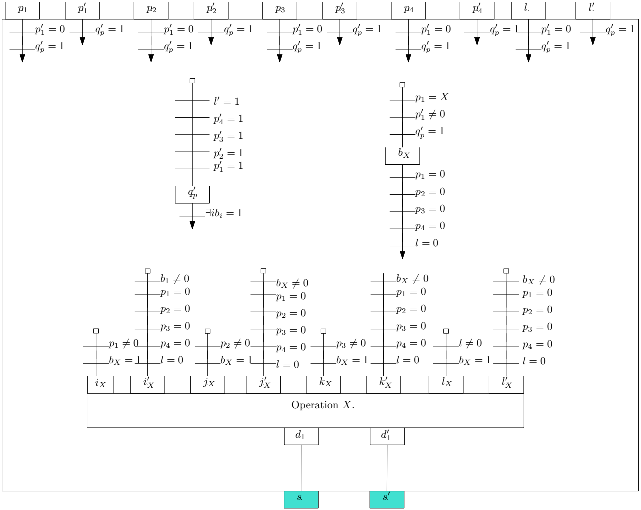
Constant step 2



Constant step 3



Operation selection step 3



Future Work

- Physical realisations.
- Equivalence to other P systems (cP, ?).
- Costs, e.g. pipes vs valves.

Questions

THANK YOU!

Any questions?